# REGIME-DEPENDENT EFFECTS OF UNCERTAINTY SHOCKS: A STRUCTURAL INTERPRETATION

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ABSTRACT. Using a Markov-switching VAR, we show that the effects of uncertainty shocks on output are four times higher in a regime of economic distress than in a tranquil regime. We then provide a structural interpretation of these facts. To do so, we develop a business cycle model, in which agents are aware of the possibility of regime changes when forming expectations. The model is estimated using a Bayesian minimum distance estimator that minimizes, over the set of structural parameters, the distance between the regime-switching VAR-based impulse response functions and those implied by the model. Our results point to worsening credit-market conditions that amplify shocks in distress periods. Finally, we show that the expectation effect of regime switching in financial conditions is an important component of the financial accelerator mechanism. If agents are more pessimistic about future financial conditions, then the output effects of shocks are amplified.

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## I. Introduction

It has been well documented that higher uncertainty reduces aggregate activity, leading to higher unemployment, and lower investment and output.[1] Recent empirical studies have also emphasized highly nonlinear effects, depending on the state of the economy; adverse effects of uncertainty shocks are greater in periods of economic distress than in tranquil periods.[2] However, little is known about the structural factors in accounting for these changes as inference of nonlinear relationships presents econometric challenges within a quantitative general equilibrium framework.

The objective of this paper is to fill part of this gap by exploring, through a novel econometric estimation, potential changes in the underlying structure of the economy that could explain such a nonlinearity. Disentangling the causes is important for understanding the extent to which economic activity can respond to future uncertainty shocks as well as the role that policy can play in order to mitigate those adverse effects.

We first reproduce the empirical evidence of highly nonlinear effects within a Markov-switching Structural Vector Autoregression (MS-SVAR) framework. We use U.S. quarterly data and include GDP growth, a measure of uncertainty (i.e., the VIX index), and a credit spread. The model identifies two distinct regimes. The first was seen in nearly all the years of episodes of high inflationary pressure in the 1970s and 1980s, during serious turbulence that marked 2001-2003 period (including the 9/11 terrorist attacks, Dot-com bubble, and corporate scandals), and during the global financial crisis. The second covers periods of tranquility. We show that, under the first regime, the adverse output effects of an increase in uncertainty appear to be four times higher than under the second regime.

We then focus on the potential explanations for this regime-dependent evidence by estimating a Markov-switching Dynamic Stochastic General Equilibrium (MS-DSGE) model with financial frictions and uncertainty shocks. Our framework is an extension of the model with asymmetric information between borrowers and lenders and costly monitoring proposed by Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2014),

[1]See, among others, Bloom (2009), Stock and Watson (2012), Glover and Levine (2015), Leduc and Liu (2016), Basu and Bundick (2017), and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018).

[2]See Caggiano, Castelnuovo, and Groshenny (2014), Caggiano, Castelnuovo, and Nodari (2017), and Alessandri and Mumtaz (2019).

by allowing key macroeconomic and financial parameters of the model to evolve over time according to a Markov-switching process. Our empirical approach is analogous to the impulse response matching approach used by Rotemberg and Woodford (1997) and Christiano, Eichenbaum, and Evans (2005), except that we are estimating the parameters to fit our regime-dependent impulse responses from a MS-SVAR, as opposed to impulse responses from a constant-parameters SVAR. To the best of our knowledge, our paper represents the first attempt to estimate a medium-scale MS-DSGE model by matching the MS-SVAR-implied impulse responses to those produced by the MS-DSGE model. We believe our MS-SVAR-implied impulse responses approach is a promising tool to infer MS-DSGE models, and can be seen as an alternative to the full Bayesian approach implemented notably by Liu, Waggoner, and Zha (2011) and Bianchi (2013).

Our estimates point out greater asymmetric information problems in the distress regime, which manifest themselves through a higher monitoring cost of defaulting borrowers, as compared to the normal regime. As a result, the optimal financial contract typically implies higher deadweight losses and an external finance premium that becomes more (negatively) sensitive to the borrower's net worth. Say it differently, the financial accelerator mechanism is stronger in the distress regime. It then becomes straightforward to understand why the response of the economy to uncertainty shocks[3] differs across regimes. Under both regimes, when uncertainty rises, banks protect themselves by raising the interest rate charged on loans to firms (i.e., external finance premium), as there are more low-productivity firms — and also more high-productivity firms, but this does not benefit to banks — and thus more default risks. It follows a decline in demand for capital, and so in net worth, investment spending and economic activity. In distress periods, higher monitoring costs cause banks to charge higher interest rates and firms to make larger cut to their investment projects, implying therefore a larger and longer-lasting decline in economic activity than in normal times.

The key insight of our MS-DSGE model is that variations in the MS-SVAR dynamics of the effects of uncertainty shocks have important effects on rational agents' expectation formation of the MS-DSGE model. Our estimates lie in the fact that agents are aware of

[3]Christiano, Motto, and Rostagno (2014) refer to risk shocks rather than uncertainty shocks. As stated in Bloom (2014), uncertainty is "a stand-in for a mixture of risk and uncertainty" in this literature. In our model, as in Christiano, Motto, and Rostagno (2014), uncertainty shocks shift the variance of the cross-sectional distribution of idiosyncratic productivity shocks.

the possibility of regime switches in the dynamics. That is, our MS-SVAR-based impulse response matching approach takes into account the fact that all agents of the MS-DSGE model know the transition probabilities and use them when forming expectations.

Under these circumstances, in any given regime, agents anticipate that uncertainty shocks may be accompanied by a switch to the other regime, altering considerably the macroeconomic outcomes. We consider how these expectation effects, using the terminology of Liu, Waggoner, and Zha (2009)[4], on a particular regime affect equilibrium in the other regime. In tranquil periods, characterized by a small degree of agency problems, agents may expect that the economy will move to the distress regime. If they are over-pessimistic, that is they over-estimate the probability of regime switching to more severe financial conditions in the future, this will amplify the contractionary effects of uncertainty shocks on aggregate activity. Conversely, an over-optimistic behavior dampens these negative effects. As a result, the expectation effects of regime shifts in financial conditions are part of the financial accelerator mechanism.

This paper proceeds as follows. Section II relates our contributions to the literature. To illustrate the possibility of nonlinearity between uncertainty and the macroeconomy, Section III provides empirical insights into how different the impact of uncertainty on aggregate activity is between distress and non-stress periods. Section IV interprets these differences in terms of an estimated DSGE model with financial frictions, in which agents form expectations on possible changes on the economy, and investigates the expectation effects of regime switching in the degree of financial frictions. Section V concludes.

## II. Literature review

This paper is related to an increasing literature that examines how uncertainty manifests itself and what their effects are on the rest of the economy.

Focusing on the United States, Bloom (2009), Stock and Watson (2012), Bekaert, Hoerova, and Duca (2013), Glover and Levine (2015), Leduc and Liu (2016), Basu and Bundick (2017), Creal and Wu (2017) and Ferrara and Guérin (2018), employ the "constant-parameters" approach to quantify the role of uncertainty on business cycle fluctuations. In particular,

[4]Liu, Waggoner, and Zha (2009) originally defined the expectation effects for monetary policy as "the difference between equilibrium outcome from a model that ignores probabilistic shifts in future policy regime and that from a model that takes into account such expected changes in regime".

all studies adopt linear SVARs and find a significant and long-lasting decrease of aggregate activity after a positive uncertainty shock. Empirical studies have been rapidly interested in their time-varying effects as events of high uncertainty did not always seem to spill over to the economy.[5]

Mumtaz and Theodoridis (2018) extend the standard approach by allowing time-varying parameters in SVARs. They emphasize the importance of taking into account shifts in the generation of uncertainty shocks. They show that, in particular, the impact of uncertainty shocks on aggregate activity has declined over time. However, the limitation of this paper to study episodes of distress, as considered herein, lies in the methodology itself — a model with smooth and drifting coefficients seems to be less suited for capturing rapid shifts in the behavior of the data as observed during distress periods. Economic or financial crises are well-known for hitting the economy instantaneously, which favors models with abrupt changes like Markov-switching models. Therefore, we follow Sims and Zha (2006) and estimate a MS-SVAR with Bayesian methods. Hubrich and Tetlow (2015) and Lhuissier (2017) also consider a MS-SVAR framework to capture regime switching in macroeconomic time series in distress periods.

Employing an alternative regime-switching method (i.e., a threshold VAR model), Caggiano, Castelnuovo, and Groshenny (2014), Caggiano, Castelnuovo, and Nodari (2017), and Alessandri and Mumtaz (2019) show that the real effects of uncertainty shocks strongly depend on the state of the economy. In particular, Alessandri and Mumtaz (2019) show that the effects depend on the state of financial markets and estimate that the impact on output is five times larger in periods of financial stress than in tranquil periods, while Caggiano, Castelnuovo, and Groshenny (2014) and Caggiano, Castelnuovo, and Nodari (2017) capture recession and expansion phases and show that uncertainty shocks are substantially more costly under recessions than under expansions. Our approach clearly differs since we assign probabilities to events and, therefore, we avoid to make the assumption that the probability of a regime switch is either one or zero. Moreover, estimating these probabilities is essential to analyze

[5]Bloom (2009) documents a variety of events that generate significant uncertainty about the future, but they are not always associated with large decline in output.

the importance of expectation effects of regime shifts in equilibrium dynamics of our MS-DSGE model, and therefore, in the transmission mechanism of uncertainty shocks to the aggregate economy.

Our analysis is related to a growing body of evidence which documents the interactions between uncertainty and financial conditions within an equilibrium business cycle framework — notable examples are Christiano, Motto, and Rostagno (2014), Gilchrist, Sim, and Zakrajšek (2014), Bloom, Alfaro, and Lin (2019), Brand, Isoré, and Tripier (2019), and Arellano, Bai, and Kehoe (2019). More specifically, our framework closely follows Christiano, Motto, and Rostagno (2014), who investigate the real role of uncertainty shocks in the context of the financial accelerator model initially developed by Bernanke, Gertler, and Gilchrist (1999). Note, however, that the severity of agency problems (i.e., monitoring costs) remains unchanged over time within their framework. Levin, Natalucci, and Zakrajšek (2004), and more recently, Lindé, Smets, and Wouters (2016) and Fuentes-Albero (2019) make it time-varying without, however, investigating the macroeconomic implications of uncertainty shocks, and the role of expectation effects of regime shifts in financial frictions in shaping the macroeconomic outcomes.

A few other papers in the literature study the origins and the effects of uncertainty shocks in empirical medium-scale DSGE Models. In particular, Bianchi, Kung, and Tirskikh (2018) examine the effects of demand-side and supply-side uncertainty through multiple endogenous risk propagation channels. We differ from this paper in that we (i) focus our structural analysis on the nonlinear effects of uncertainty shocks, (ii) emphasize a key role in financial conditions, (iii) propose a novel econometric estimation based on impulse response matching approach, and (iv) do not allow for various risk channels.

Our paper is also related to an increasing literature investigating the importance of expectation effects in regime shifts in a Markov-switching framework. This concept was originally defined by Liu, Waggoner, and Zha (2009) in the context of regime changes in monetary policy, and then have been extensively studied thereafter. Bianchi (2013) considers "beliefs counterfactuals" to quantify the importance of expectation effects in business cycle fluctuations. Foerster (2016) distinguishes the expectation effects of regime switching in the inflation target from those in the inflation response. Bianchi and Melosi (2016) develop a Bayesian learning process about regime shifts that influence the expectations formation mechanism.

Bianchi and Ilut (2017) consider expectation effects in monetary/fiscal policy mix changes. We extend this concept and apply it for regime shifts in the degree of financial frictions. Interestingly, the expectation effects embedded in our model share some features with the anticipation effect described by He and Krishnamurthy (2019) in the context of a model with occasionally binding financial constraints. In their model, financial constraints have effects on the equilibrium even when they are not binding (which corresponds to the tranquil regime in our model) because agents anticipate that they may bind in the future (which corresponds to the realization of the stress regime in our model).

From a methodological standpoint, this paper is related to an increasing literature dealing with estimation and simulation of DSGE models in which stochastic volatilities and structural parameters are allowed to follow Markov-switching processes. This literature includes, among others, Liu, Waggoner, and Zha (2011), Bianchi (2013), Davig and Doh (2014), Lhuissier and Zabelina (2015), Bianchi and Melosi (2017), and Lhuissier (2018). The standard approach for inference of MS-DSGE models employed by all of these papers is to build the state-space representation of the MS-DSGE models adapted from the the standard Kim and Nelson (1999)'s filter. In contrast, our approach dispenses with such a filter as inference is directly done by minimizing the gap between theoretical and empirical impulse response functions.

## III. Evidence of time variation in the effects of uncertainty shocks

This section documents changes in the effects of uncertainty shocks on aggregate activity over time by employing a Markov-switching framework.

### III.1. **Markov-switching Structural Bayesian VARs.**

Following Hamilton (1989), Sims and Zha (2006), and Sims, Waggoner, and Zha (2008), we employ a Markov-switching Bayesian structural VAR model of the following form:

$$y_t' A(s_t^{\mathrm{c}}) = \sum_{i=1}^{\rho} y_{t-i}' A_i(s_t^{\mathrm{c}}) + C(s_t^{\mathrm{c}}) + \varepsilon_t' \Xi^{-1}(s_t^{\mathrm{v}}), \qquad t = 1, \ldots, T, \tag{1}$$

where $y_t$ is defined as $y_t \equiv [gdp_t, vix_t, sp_t]'$; $gdp_t$ is the logarithm of U.S. real Gross Domestic Product (GDP); $vix_t$ is the VIX index, a proxy for uncertainty; and $sp_t$ is the BAA-AAA credit spread. Data sources are presented in the Online Appendix. The overall sample period is 1962:Q3 to 2018:Q2. We set the lag order to $\rho = 2$. Our parsimonious specification is justified by the fact that it becomes quickly challenging to estimate Bayesian MS-SVAR models as the number of observables and lags grows. Note also that this is in line with the literature that allows for time-varying parameters in VARs (e.g., Primiceri, 2005; Cogley and Sargent, 2005; Bianchi and Melosi, 2017).

We assume a two-regimes process governing equation coefficients and constants ($s_t^{\text{c}}$), and a three-regimes process governing disturbance variances ($s_t^{\text{v}}$). The regimes evolve according to two transition matrices as follows:

$$Q^{\text{c}} = \begin{bmatrix} q_{1,1}^{\text{c}} & q_{1,2}^{\text{c}} \\ q_{2,1}^{\text{c}} & q_{2,2}^{\text{c}} \end{bmatrix}, \qquad \text{and} \qquad Q^{\text{v}} = \begin{bmatrix} q_{1,1}^{\text{v}} & (1-q_{2,2}^{\text{v}})/2 & 0 \\ 1-q_{1,1}^{\text{v}} & q_{2,2}^{\text{v}} & 1-q_{3,3}^{\text{v}} \\ 0 & (1-q_{2,2}^{\text{v}})/2 & q_{3,3}^{\text{v}} \end{bmatrix}. \tag{2}$$

The restricted transition matrix $Q^{\text{v}}$ implies that when we are in regime $j$, we can only move to regime $j-1$ or $j+1$. Sims, Waggoner, and Zha (2008) argue that such a restriction tends to fit the macroeconomic data better.

We assume that $\varepsilon_t$ follows the following distribution:

$$p(\varepsilon_t) = \text{normal}(\varepsilon_t | 0_n, I_n), \tag{3}$$

where $0_n$ denotes an $n \times 1$ vector of zeros, $I_n$ denotes the $n \times n$ identity matrix, and normal$(x|\mu, \Sigma)$ denotes the multivariate normal distribution of $x$ with mean $\mu$ and variance $\Sigma$. Finally, $T$ is the sample size; $A(s_t)$ is a $n$-dimensional invertible matrix under the regime $s_t$; $A_i(s_t)$ is a $n$-dimensional matrix that contains the coefficients at the lag $i$ and the regime $s_t$; $C(s_t)$ contains the constant terms; and $\Xi(s_t)$ is a $n$-dimensional diagonal matrix.

Following Sims and Zha (1998), we exploit the idea of a Litterman's random-walk prior to structural-form parameters.[6] The Online Appendix provides the details techniques for the Sims and Zha (1998) prior.

Finally, the prior duration of each regime is about five quarters. We have also used other prior duration and the main conclusions remain unchanged.

[6]Regarding the Sims and Zha (1998) prior, the hyperparameters are defined as follows: $\mu_1 = 1.00$ (overall tightness of the random walk prior); $\mu_2 = 1.00$ (relative tightness of the random walk prior on the lagged parameters); $\mu_3 = 0.1$ (relative tightness of the random walk prior on the constant term); $\mu_4 = 1.0$ (erratic sampling effects on lag coefficients); $\mu_5 = 0.0$ (belief about unit roots); and $\mu_6 = 0.0$ (belief in cointegration relationships). To match the usual interpretation of Litterman's prior on the reduced form, we drop the two kinds of true dummy observations ($\mu_5$ and $\mu_6$) introduced by Sims and Zha (1998). See also Doan, Litterman, and Sims (1984) and Sims (1993).

III.2. **Identification.** We identify uncertainty shocks by combining two kinds of restriction. The first is based on traditional sign restrictions on the impulse response functions, as developed by Faust (1998), Canova and Nicolo (2002), and Uhlig (2005). We impose that an uncertainty shock induces a simultaneous rise in the VIX index and credit spread. The argument for this restriction is based on the idea that increases in financial uncertainty is frequently associated with significant increases in credit spreads, as shown in Stock and Watson (2012). We also assume that innovations to uncertainty cause an immediate fall in output. This restriction is motivated by the large theoretical literature views that uncertainty has recessionary effects. See Bloom (2014) for a survey of the literature.

The above restriction is not sufficient to guarantee *pure* uncertainty shocks due to the high degree of comovement between the uncertainty proxy and credit spread (e.g., Stock and Watson, 2012). It might be possible that shocks originating from financial sector are present into uncertainty shocks. The second kind of restriction allows us to completely disentangle between these two types of shock. We use a criterion that impose a restriction on the one-step ahead prediction error variance of our uncertainty variable. We impose the restriction that the uncertainty shock is the overwhelming driver of the unexpected movement in the VIX index, i.e., it explains, at least, 50 percent of variations in the VIX index. This kind of restriction is in line with Caldara, Fuentes-Albero, Gilchrist, and Zakrajšek (2016) who identify uncertainty shocks as innovations explaining the maximum amount of variability in an uncertainty indicator to disentangle them from financial shocks.

By combining the appeal of forecast error variance restrictions approach with the advantages of sign restrictions, we are able to isolate fluctuations in uncertainty and its effects on economic activity.

III.3. **Empirical results.** In this section, we report our main empirical results produced by the MS-SVAR model. First, we present, in Section III.3.1, the posterior distribution of the estimated model. We then report, in Section III.3.2 the historical evolution of uncertainty for each variable. Finally, impulse responses of endogenous variables to uncertainty shock are reported in Section III.3.3. The results shown are based on 10 million draws with the Gibbs sampling procedure (see the Online Appendix for details). We discard the first $1,000,000$ draws as burn-in, then keep every 100th draw.

III.3.1. *Posterior distribution.* In this section, we present key results produced from the model. Figures 1 and 2 show the probabilities of being in a specific regime for each process ($s_t^v$ and $s_t^c$) over time. The probabilities are smoothed in the sense of Kim (1994); i.e., full sample information is used in getting the regime probabilities at each date.

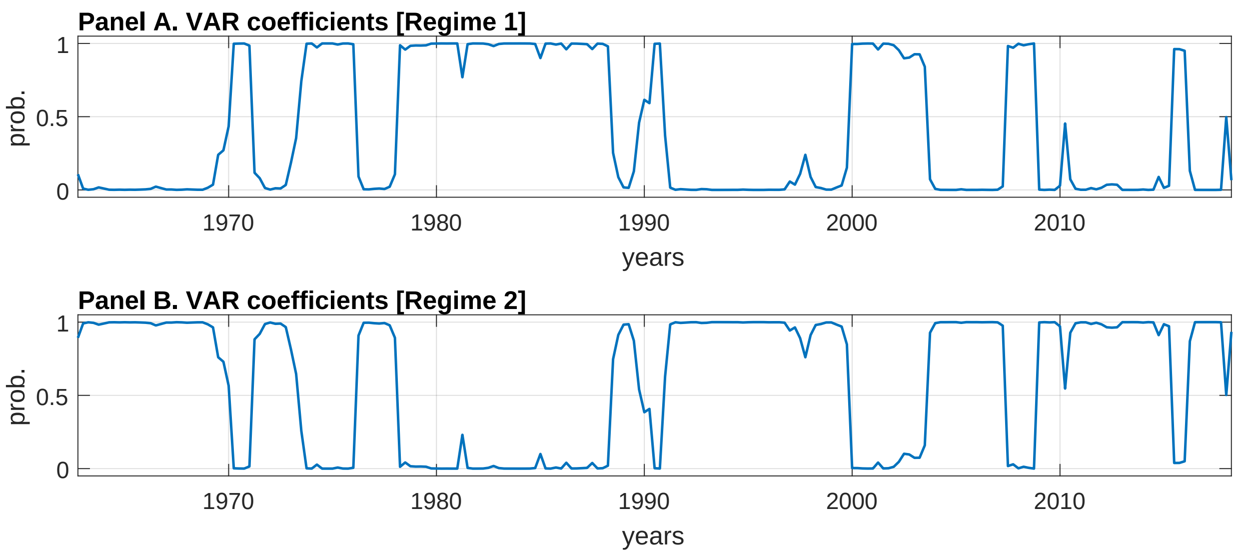


FIGURE 1. Sample period: 1962.Q4-2018.Q2. State Probabilities (Two-State Structural Coefficients).

When looking at the process in which equation coefficients are allowed to change (see $s_t^c$ shown in Figure 1), it is apparent that Regime 1 ($s_t^c = 1$) was prevailing during episodes of high inflationary pressure in the 1970s and 1980s, and dominant during the age of the 9/11 attacks, Dot-com bubble, and corporate scandals. This regime was also in place during the financial crisis originated by subprime mortgages, as well as during the European debt crisis. We thus label this regime as the *distress* regime. All of the above-mentioned sub-periods, captured by this regime, contain the same similarities, namely major disruption in financial markets, macroeconomic imbalances, and heightened uncertainty. Regime 2 has prevailed for the remaining years of the sample, characterized by episodes of tranquility. We label it as the *tranquil* regime. Regarding the process governing the structural disturbance variances, $s_t^v$, the model clearly captures three distinct regimes of volatility: a *low-*, *high-*, and *extreme-*volatility regime, as shown in Table 1.[7] Looking at Figure 2, the high-volatility regime (i.e.,

[7]Following Sims and Zha (2006), we normalize the size of shock variances to unity in Regime 1, $s_t^v = 1$.

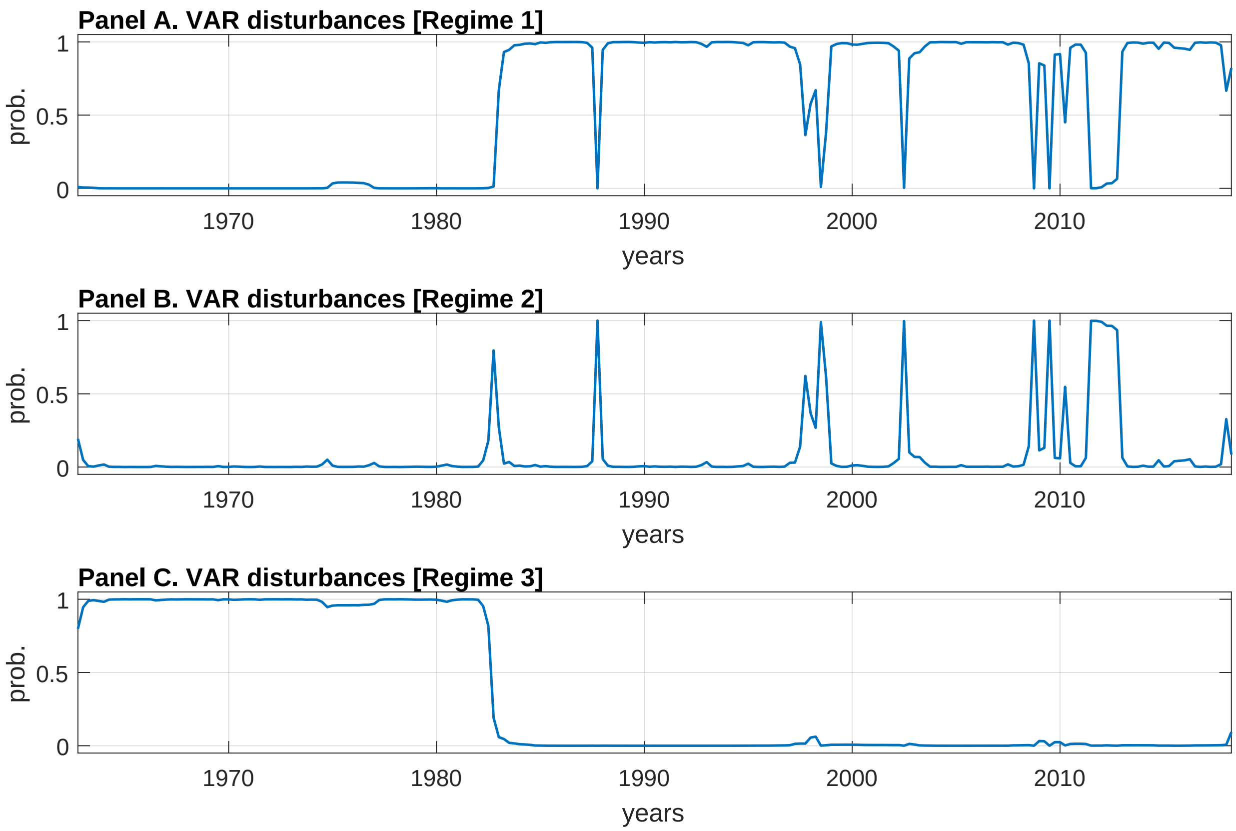


FIGURE 2. Sample period: 1962.Q4-2018.Q2. State Probabilities (Three-State Disturbance Variances).

Regime 3) corresponds clearly to the *pre*-Great Moderation period, where the size of shock variance in output is relatively twice larger than that experienced in the low-volatility regime (i.e., Regime 1). The higher degree of volatility in the pre-1980s period corroborates, for example, with Kim and Nelson (1999). Finally, the extreme-volatility regime (i.e., Regime 2) identifies exceptional events, like the beginning of the Great Recession in 2008.

Table 2 reports the estimated transition matrices at the posterior mode, with 68% probability intervals in brackets, for both Markov-switching processes. Looking at the $s_t^c$ process, the distress regime ($q_{11}^{\mathrm{c}} = 0.8969$) is slightly less persistent (an average duration of about 9 quarters) than the tranquil regime ($q_{22}^{\mathrm{c}} = 0.9325$) with an average duration over 15 quarters. Looking at the $s_t^v$ process, Regimes 1 and 3 are unsurprisingly the most persistent, with $q_{11}^{\mathrm{v}} = 0.9432$ and $q_{33}^{\mathrm{v}} = 0.9846$, respectively. Regime 2, has a very short-lived duration of about 3 quarters. The tight interval probabilities reinforce the credibility of the estimated mode values.

TABLE 1. Relative shock standard deviations across regimes

| | Production gdp | Uncertainty vix | Financial sp |
|---|---|---|---|
| $s_t^{\mathrm{v}} = 1$ | 1.0000 [1.0000;1.0000] | 1.0000 [1.0000;1.0000] | 1.0000 [1.0000;1.0000] |
| $s_t^{\mathrm{v}} = 2$ | 1.4270 [1.2750;2.2982] | 5.6380 [3.7603;6.4075] | 3.3247 [1.9506;3.7224] |
| $s_t^{\mathrm{v}} = 3$ | 2.0527 [1.8723;2.4634] | 1.5571 [1.2646;1.7623] | 1.1065 [0.9246;1.2625] |

*Note:* Posterior modes and [16th , 64th] percentiles are reported.

TABLE 2. Estimated transition matrices

| VAR disturbances | | | | VAR coefficients | | |
|---|---|---|---|---|---|---|
| | $s_t^{\mathrm{v}} = 1$ | $s_t^{\mathrm{v}} = 2$ | $s_t^{\mathrm{v}} = 3$ | | $s_t^{\mathrm{c}} = 1$ | $s_t^{\mathrm{c}} = 2$ |
| $s_t^{\mathrm{v}} = 1$ | 0.9432 [0.9178;0.9717] | 0.1828 [0.0869;0.2066] | 0.0000 [0.0000;0.0000] | $s_t^{\mathrm{c}} = 1$ | 0.8969 [0.7754;0.8980] | 0.0675 [0.0536;0.1215] |
| $s_t^{\mathrm{v}} = 2$ | 0.0568 [0.0283;0.0822] | 0.6345 [0.5869;0.8261] | 0.0154 [0.0157;0.0660] | $s_t^{\mathrm{c}} = 2$ | 0.1031 [0.1020;0.2246] | 0.9325 [0.8785;0.9464] |
| $s_t^{\mathrm{v}} = 3$ | 0.0000 [0.0000;0.0000] | 0.1828 [0.0869;0.2066] | 0.9846 [0.9340;0.9843] | | | |

*Note:* Posterior modes and [16th , 64th] percentiles are reported.

In summary, our results suggest that the economy has experienced shocks whose the size change over time. Interestingly, the behavior of the economy — characterized by the systematic part of the model, i.e., equation coefficients — is different in distress periods than in tranquil periods. The objective of the next sections is then to investigate the extent at which economic dynamics differ across regimes.

III.3.2. *Historical evolution of uncertainty.* Given the importance we attach to uncertainty fluctuations in this paper, we characterize agents' uncertainty using our MS-SVAR model. Bianchi (2016) describes how to characterize uncertainty in presence of regime changes in multivariate models. Uncertainty computed in this way reflects all sources of uncertainty faced by an agent: the possibility of regime changes, uncertainty around the state of the economy, uncertainty about the regime in place, and the possibility of Gaussian shocks.

Following Bianchi (2016)'s methodology, we measure uncertainty of each endogenous variable $y_t$ at horizon $s$ by its conditional standard deviation.

Figure 3 reports the evolution of uncertainty at each point in time. The time horizon goes from one quarter, blue line, to five years, green line. Not surprisingly, the volatility regime that is in place affects the evolution of uncertainty. For example, prior to the mid 1980's, the level of uncertainty for the VIX index turns out to be lower than during the post mid 1980's. While short run uncertainty for output is larger than its long run uncertainty, this is not always the case for VIX index and the premium. Periods of high volatility imply that a relative higher short run uncertainty. This is because, as the time horizon increases, the probability of still being in the high volatility regime decreases. Finally, times of recession are remarkably well associated with high output uncertainty. This is in line with the literature, see for example Bloom (2014).

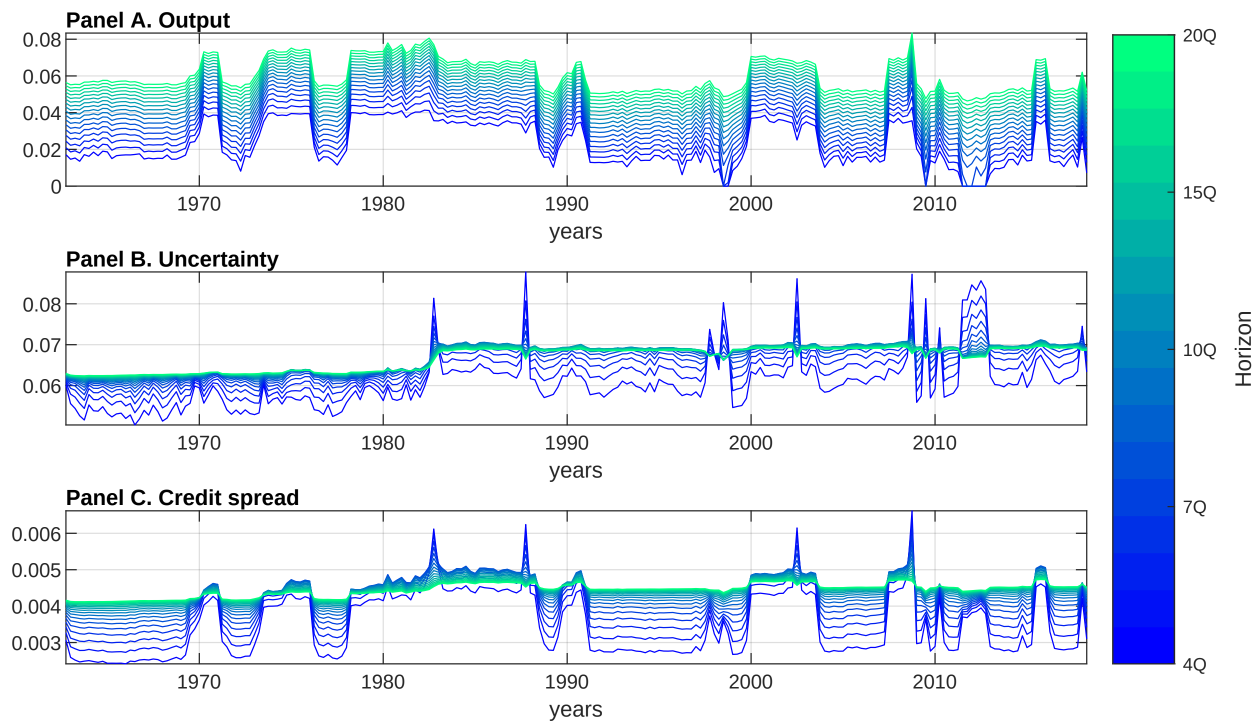


FIGURE 3. Historical evolution of volatility for output, uncertainty, and credit spread for horizons going from 1 year to 5 years. The blue lines correspond to the short horizons, while the green lines correspond to the long horizons.

III.3.3. *Regime-dependent dynamic effects of uncertainty shocks.* We illustrate possible differences in dynamics across the two regimes of the process governing equation coefficients,

$s_t^c$, by examining the conditional response of the rest of the economy to a pure disturbance in uncertainty ("one-time uncertainty shock").[8]

Figure 4 reports the impulse responses of endogenous variables across the two regimes. The first column shows the responses in the tranquil regime, while the responses in the stress regime are displayed in the second column. All of these panels display the deviation in percent for the series entered in log-levels (output), whereas it displays the deviation in percentage points for the VIX index and credit spread. The third column shows the differences between impulse responses of the two regimes. In any column, the solid lines represent the median, with the 68% and 90% probability intervals displayed in dotted lines. For comparability across regimes, our uncertainty shock is scaled to induce a 10 percentage points immediate increase in the VIX index.

Looking at this figure, the responses of our measure of aggregate activity do vary much over time, indicating that the differences among the two regimes in the coefficients of the system of equations are very large. After a positive innovation in our uncertainty measure that causes a 10 percentage points increase in the VIX index, the output falls slowly and moderately in the tranquil regime, but falls quickly and considerably in the distress regime, until reaching its minimum after 3 quarters (at this point, the impact turns out to be four times higher in the distress regime). These differences appear to be robust when taking into account the 68 percent probability intervals (right-top panel); error bands of the differences lie exclusively within the negative region over the first 5 quarters.

Interestingly, the response of credit spread is much larger in the distress regime, indicating credit costs for firms are relatively high. Once again, error bands reinforce these results. We thus might say that the amplification effects on output occur primarily through changes in

[8]Here, we assume that a particular regime will last in the wake of the shock, although agents take into account the possibility of regime shifts. Alternatively, we could have employed the generalized impulse-response function (GIRF) developed by Koop, Pesaran, and Potter (1996), and transposed to MS-SVAR models by Karamé (2015) and Bianchi (2016). GIRF makes allowance for the dependence on initial conditions, future shocks, and future regimes. Our choice for conditional impulse responses is justified by the fact that we want to highlight the regime-specific features and more specifically to understand how the economy behaves if a specific regime will prevail over the relevant horizon.

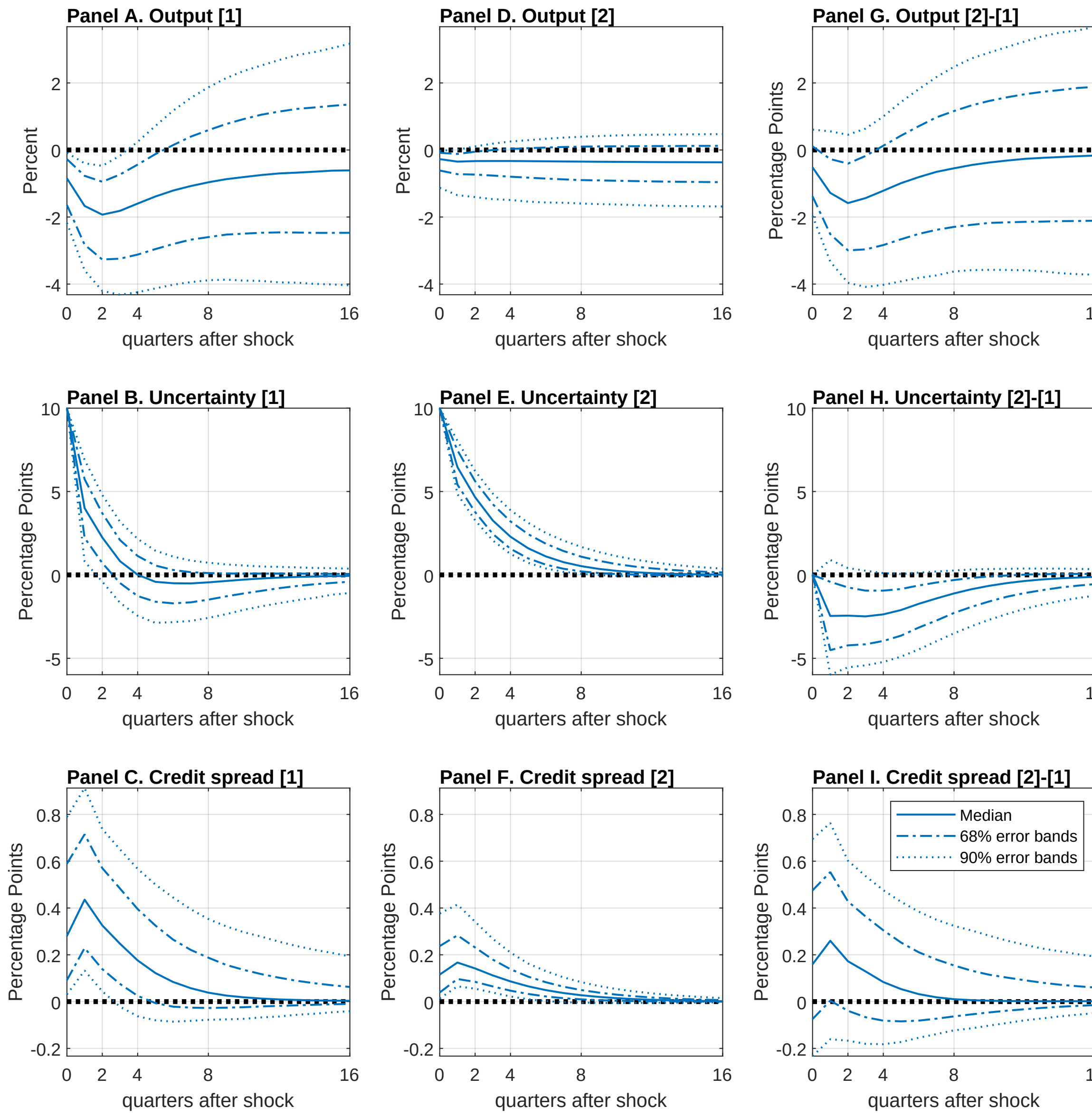


FIGURE 4. Impulse-response functions to uncertainty shock under both regimes obtained from the identified MS-SVAR model. The first and second columns report impulse responses of endogenous variables under distress and tranquil regimes, respectively. The last column displays the difference between the two regimes. In each case, the median is reported in solid line and the 68% and 90% error bands in dotted lines.

credit spreads. We investigate this intuition in the next section through inference of a MS-DSGE model by using the regime-dependent impulse responses obtained from the identified MS-SVAR model.

III.3.4. *Robustness.* In order to assess the robustness of results, we study a number of alternative identification schemes. First, we change the threshold level of minimum contribution of the shock to the prediction error variance in the VIX index. Second, we relax the assumption that restrictions are only imposed within quarter. Third, we employ an alternative restriction on forecast error variance. The results of this section are available in the Online Appendix.

**(1) The threshold of minimum contribution.** In our benchmark identification, we impose that the uncertainty shock is the overwhelming driver of the unexpected movement in the VIX index, i.e., it explains at least 50% of uncertainty variations. Several other thresholds were examined to determine if they deliver different outcomes. The levels of alternative threshold are (1) 60%; (2) 70%; and (3) 80%. Clearly, the changes in the threshold level do not affect the main conclusions. Impulse responses are close to those reported in the previous section.

**(2) Alternative restrictions horizon.** Our both types of restrictions are only imposed within quarter in the baseline identification. In this section, we reinforce this assumption and assume now that those restrictions are also imposed for the next two quarters. By doing so, we guarantee that uncertainty shocks generate a persistent increase in the VIX index, rather than just a one-off spike in volatility. We find that economic implications produced from this scheme remain unchanged.

**(3) Alternative restriction on forecast error variance.** Our restriction imposed on forecast error variance is similar in some way to the identification strategy used by Uhlig (2003) who identifies a structural shock by searching for the shock which has the larger contribution compared to the largest contribution of any other shock. By contrast, we identify a shock by searching for the one whose its contribution is larger than the sum of contributions of all other shocks. One may ask how sensitive the results are with the traditional Uhlig (2003)'s method. We have done this exercise and clearly, regime-dependent impulse responses remain similar.

## IV. A structural interpretation

Now that we have shown that there are important changes in the transmission mechanism of uncertainty shocks across economic regimes, we use a microfounded business cycle model to provide a structural interpretation of these changes. Section IV.1 and IV.2 present the microfounded model and the solving method, respectively. The estimation results are reported in Section IV.3.

IV.1. **Model.** We use a medium-scale DSGE model along the lines of Christiano, Motto, and Rostagno (2014), but with two important differences. First, we assume only one source of perturbations in the economy, namely uncertainty shocks, defined in the model as fluctuations in the volatility of cross-sectional idiosyncratic uncertainty. Second, some structural parameters are allowed to vary over time according to a two-states, first-order Markov-switching process, $\chi_t$, with transition matrix $P = (p_{ij})_{(i,j \in \{1,2\})}$, where $p_{ij}$ denotes transition probabilities. To maintain model tractability, we do not allow all structural parameters to change over time. We assume that there are three set of candidates for explaining the differences in economic dynamics across regimes. The first is *real*, captured by the functional form of investment adjustment costs. The second is *nominal*, represented by the parameters describing the monetary policy reaction function and the parameters characterizing the pricing behavior of firms and workers. The third is *financial*, represented by parameters governing the degree of agency problems between borrowers and lenders within the financial contract. The following sections present a complete description of the optimization problems solved by firms, households, and entrepreneurs.

IV.1.1. *Producers.* The final good sector is perfectly competitive. The representative final good producer combines intermediate goods $Y_{j,t}$ to produce a homogeneous good $Y_t$ using the following technology:

$$Y_t = \left[ \int_0^1 Y_{j,t}^{\frac{1}{\lambda_f}} dj \right]^{\lambda_f}, \qquad 1 \leq \lambda_f < \infty, \tag{4}$$

where $\lambda_f$ is the elasticity of substitution among intermediate goods. Monopolistic producers, indexed by $j$, demand capital and labor in order to maximize their cost of production subject to the demand function for their good using the following production technology:

$$Y_{j,t} = (u_t K_{j,t})^{\alpha} (z_t l_{j,t})^{1-\alpha} - \varphi z_t^*, \qquad 0 < \alpha < 1, \tag{5}$$

where $u_t$ is the utilization rate of capital, $\varphi$ a fixed cost of production, and $z_t^*$ is a combination of the labor productivity trend, $z_t$, and the investment-specific technology trend, $\Upsilon^t$, as follows: $z_t^* = z_t \Upsilon^{(\frac{\alpha}{1-\alpha})t}$. Also, $K_{j,t}$ is the services of effective capital and $l_{j,t}$ is the quantity of homogeneous labor hired by the $j$th intermediate good producer.

The monopoly supplier of $Y_{j,t}$ sets its price, $P_{j,t}$, subject to nominal rigidity. At each period, a randomly selected fraction of intermediate good firms, $1 - \xi_p(\chi_t)$, can reset their price while the complementary fraction follow a simple rule of thumb, $P_{j,t} = \tilde{\pi}_t P_{j,t-1}$, where $\tilde{\pi}_t \equiv (\pi^{target})^{\iota_p(\chi_t)}(\pi_{t-1})^{1-\iota_p(\chi_t)}$, with $\pi_{t-1} \equiv P_{t-1}/P_{t-2}$, and $\pi^{target}$ is the target inflation rate of the monetary authority. Note that the pricing process is allowed for regime changes.

IV.1.2. *Households.* There is a large number of identical and competitive households. We assume that each household contains every type of differentiated labor, $h_{i,t}, i \in [0,1]$. Each household has a large number of entrepreneurs, but we defer the description of these agents to the next subsection. The preferences of the representative household maximizes the expected discounted sum of utilities given by

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log\left(C_t - bC_{t-1}\right) - \psi_L \int_0^1 \frac{h_{i,t}^{1+\sigma_L}}{1+\sigma_L} di \right\}, \tag{6}$$

subject to the law of capital accumulation

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}, \chi_t\right)\right] I_t, \tag{7}$$

and the budget constraint

$$R_t B_t + (1-\tau^l) \int_0^1 W_{i,t} h_{i,t} di + Q_{\bar{K},t}\bar{K}_{t+1} + \Pi_t = B_{t+1} + (1+\tau^c) P_t C_t + Q_{\bar{K},t}(1-\delta)\bar{K}_t + \frac{P_t}{\Upsilon^t} I_t, \tag{8}$$

where $C_t$ stands for consumption, $P_t$ the price of consumption, $h_{i,t}$ hours worked, $\bar{K}_t$ raw capital, $Q_{\bar{K},t}$ the price of capital, $I_t$ investment, $B_t$ one-period bonds, $R_t$ the nominal interest rate on these bonds, $W_{i,t}$ the wages of differentiated labor, and $\Pi_t$ the firms' profits. Households preferences are determined by $\beta$ the discount factor, $b$ the degree of habit formation, $\psi_L$ and $\sigma_L$ which determines labor supply elasticity. The parameters $\tau^c$ and $\tau^l$ are consumption and labor tax rates. Physical capital depreciates at rate $\delta$. The investment adjustment cost function is subject to the Markov-switching process according to the following specification:

$$S\left(x_t, \chi_t\right) = \frac{1}{2}\left[\exp\left[\sqrt{S''(\chi_t)}(x_t - \bar{x})\right] + \exp\left[-\sqrt{S''(\chi_t)}(x_t - \bar{x})\right] - 2\right], \tag{9}$$

where $x_t \equiv (I_t/I_{t-1})$, which the steady-state level is $x$. Note that $S(x,\chi_t) = S'(x,\chi_t) = 0$ and the curvature parameter $S''(x,\chi_t) = S''(\chi_t)$ changes across regimes.

Households' differentiated labor services are aggregated by a "labor contractor" into homogeneous labor supply, $l_t$, as

$$l_t = \left[\int_0^1 (h_{i,t})^{\frac{1}{\lambda_w}} di\right]^{\lambda_w}, \tag{10}$$

with $\lambda_w \geq 1$ the elasticity of substitution of the $i$ labor types. This homogeneous labor is sold to monoplistic producers at wage $W_t$ whereas each worker's type $i$ is paid a wage $W_{i,t}$. Therefore, the contractor chooses the quantity of each labor $i$ type, $h_{i,t}$, such that

$$\max_{h_{i,t}} W_t l_t - \int_0^1 W_{i,t} h_{i,t} di. \tag{11}$$

The wage rate for each labor type $W_{i,t}$ is subject to nominal rigidity. At each period, a randomly selected subset of $1-\xi_w(\chi_t)$ contractors can change their wage optimally, while the other follows an indexation rule based on productivity growth rate, wage inflation, and past wage as follows: $W_{i,t} = \mu_z^* \tilde{\pi}_{w,t} W_{i,t-1}$. Here $\mu_z^*$ denotes the growth rate of $z_t^*$ in nonstochastic steady state, and $\tilde{\pi}_{w,t} \equiv (\pi^{target})^{\iota_w(\chi_t)}(\pi_{t-1})^{1-\iota_w(\chi_t)}$. Note that the wage pricing process is allowed for regime changes.

IV.1.3. *Financial contract.* In each period of time, an exogenous fraction $(1-\gamma)$ of entrepreneurs dies and is born. Each $N$-type entrepreneur is endowed with a personal wealth $N$ and the aggregate wealth is $N_{t+1} = \int_0^\infty N f_t(N)\, dN$, where $f_t(N)$ is the density function of entrepreneurs with wealth $N$.

At the end of period $t$, each $N$-type entrepreneur chooses individual capital holding $\bar{K}_{t+1}^N$ for the next period, bought at market price $Q_{\bar{K},t}$, taken as given, from households. This capital purchase is made using the personal wealth $N$ and a one-period debt amount $B_{t+1}^N$ contracted optimally with the bank at the end of time $t$. Thus, we obtain the following constraint

$$Q_{\bar{K},t}\bar{K}_{t+1}^N = N + B_{t+1}^N. \tag{12}$$

This $\bar{K}_{t+1}^N$ "raw" capital is then transformed into $\omega\bar{K}_{t+1}^N$ "efficiency" units, where $\omega$ has a unit-mean log-normal distribution that is drawn independently across time and across entrepreneurs. The standard deviation of $\log \omega$, denoted $\sigma_t$, captures the idiosyncratic uncertainty in actual business activities and follows an exogenous stochastic process as

$$\log \sigma_t = (1 - \rho_\sigma(\chi_t)) \log \sigma + \rho_\sigma(\chi_t) \log \sigma_{t-1} + \varepsilon_{\sigma,t}, \tag{13}$$

with $\sigma$ its steady-state value and

$$\varepsilon_{\sigma,t} = \text{normal}(\varepsilon_{\sigma,t}|0, \sigma_\sigma(\chi_t)). \tag{14}$$

The uncertainty shock, $\sigma_t$, captures the extent of cross-sectional dispersion in $\omega$. As can be seen, the persistence and the variance of the shock, $\rho_\sigma(\chi_t)$ and $\sigma_\sigma(\chi_t)$ respectively, are subject to the Markov-switching process.

Once the idiosyncratic productivity shock is realized, each $N$-type entrepreneur determines the utilization rate, $u_t^N$, of its effective capital, and then supplies its amount of capital services, $u_t^N \omega \bar{K}_t^N$, at a market rental rate, $\tau^k$. Then, each $N$-type entrepreneur is left, after depreciation, with $(1-\delta)\omega K_t^N$ units of capital, which is sold to households at the price $Q_{\bar{K},t}$. In consequence, an $N$-type entrepreneur enjoys rate of return $\omega R_{t+1}$ at $t+1$, where

$$R_{t+1}^k \equiv \frac{\left(1-\tau^k\right)\left(u_{t+1} r_{t+1}^k - a(u_{t+1})\right) P_{t+1}/\Upsilon^{t+1} + (1-\delta) Q_{K,t+1} + \tau^k \delta Q_{\bar{K},t}}{Q_{\bar{K},t}}, \tag{15}$$

where $\tau^k$ denotes the tax rate on capital income, $a(u_t)$ is the capital utilization cost defined as $a\left(u_t\right) = \frac{r^k\left(\exp(\sigma_a(u_t-1))-1\right)}{\sigma_a}$, with $\sigma_a > 0$, and $r^k$ is the steady-state rental rate of capital.

The financial intermediary borrows from households at the short-term risk-free rate $R_t$ the amount $B_{t+1}$ which is provided to the entrepreneur as a one-period loan $B_{t+1}$ at interest rate $Z_{t+1}$. According to a costly-state verification loan contract, the $N$-type entrepreneur can either (i) repay the loan $B_{t+1}^N$ with state-contingent (gross) interest rate $Z_{t+1}$, or (ii) default on the loan, in which case the bank seizes a fraction $(1-\mu(\chi_t))$ of the entrepreneur's assets, where $\mu(\chi_t)$ denotes the monitoring costs. As emphasized in Carlstrom and Fuerst (1997), monitoring technology can be viewed as bankruptcy costs, and more broadly as liquidation costs since the firm being closed and its assets being liquidated. This technology is allow to vary across regimes.

There is a threshold value $\overline{\omega}$ such that an $N$-type entrepreneur pays back the loan if $\omega > \overline{\omega}_{t+1}$, and default otherwise, i.e such that

$$R_{t+1}^k \overline{\omega}_{t+1} Q_{\bar{K},t} \bar{K}_{t+1}^N = B_{t+1}^N Z_{t+1}. \tag{16}$$

An $N$-type entrepreneur values a particular debt contract according the expected net worth in period $t+1$:

$$\max_{\overline{\omega}_{t+1},L_t} \mathrm{E}_t \left\{ \left[1 - \Gamma_t\left(\overline{\omega}_{t+1}\right)\right] R^k_{t+1} L_t N \right\}, \tag{17}$$

where $\Gamma_t(\overline{\omega}_{t+1})$ the bank's share of entrepreneurial earnings, defined as

$$\Gamma_t\left(\overline{\omega}_{t+1}\right) \equiv \left[1 - F_t\left(\overline{\omega}_{t+1}\right)\right] \overline{\omega}_{t+1} + G_t\left(\overline{\omega}_{t+1}\right), \tag{18}$$

with $G_t\left(\overline{\omega}_{t+1}\right) \equiv \int_0^{\overline{\omega}_{t+1}} \omega dF_t\left(\omega\right),\ F_t(\omega) \equiv F(\omega_{t+1}, \sigma_t)$ is the cumulative distribution function of $\omega$, and the leverage ratio is defined as $L_t = Q_{\bar{K},t} K^N_{t+1}/N$, subject to the following bank participation constraint

$$\left[1 - F_t\left(\overline{\omega}_{t+1}\right)\right] Z_{t+1} B_{t+1} + \left[1 - \mu\left(\chi_t\right)\right] \int_0^{\overline{\omega}_{t+1}} \omega dF_t\left(\omega\right) R^k_{t+1} Q_{\bar{K},t} \bar{K}_{t+1} \geq B_{t+1} R_t, \tag{19}$$

and the default condition (16).

IV.1.4. *Aggregation.* We aggregate the quantity of raw capital purchased by entrepreneurs and the quantity of debt extend to entrepreneurs in period $t$ as follows

$$\bar{K}_{t+1} = \int_0^\infty \bar{K}^N_{t+1} f_t(N) dN, \qquad \text{and} \qquad B_{t+1} = \int_0^\infty B^N_{t+1} f_t(N) dN. \tag{20}$$

Also, the aggregate supply of capital services, $K_t = u_t \bar{K}_t$, must equal to the corresponding demand, $\int_0^1 K_{j,t} dj$, by the intermediate good producers.

Finally, at the aggregate level, the net worth at the end period $t$ is given by

$$N_{t+1} = \gamma \left[1 - \Gamma_{t-1}(\overline{\omega}_t)\right] R^k_t Q_{K,t-1} K_t + W^e_t, \tag{21}$$

where $W^e_t$ is a transfer from households to entrepreneurs.

IV.1.5. *Monetary authority and resource constraint.* The monetary policy rule is defined as follows:

$$R_t - R = \rho_p\left(R_{t-1} - R\right) + \left(1 - \rho_p\right) \left[ \phi_\pi(\chi_t)\left(\pi_{t+1} - \pi^*_t\right) + \frac{\phi_y(\chi_t)}{4}\left(g_{y,t} - \mu^*_z\right) \right], \tag{22}$$

where $g_{y,t}$ denotes the quarterly growth in GDP, $GDP_t$, with $GDP_t = G_t + C_t + I_t \Upsilon^{-t}$; where $G_t$ is government consumption and $\eta_g$ denotes its share in $GDP$. Note that the elasticities of the nominal rate to inflation gap and the output growth gap, respectively $\phi_\pi(\chi_t)$ and $\phi_y(\chi_t)$

are subject to the Markov-switching process. Finally, the resource constraint of the economy is

$$Y_t = C_t + \frac{I_t}{\Upsilon^t} + G_t + a\left(u_t\right)\frac{\bar{K}_t}{\Upsilon^t} + \frac{\mu(\chi_t)G\left(\overline{\omega}_t\right)R_t^k Q_{\bar{K},t-1}\bar{K}_t}{P_t}, \tag{23}$$

taking into account the costs of capital utilization and the resources used for monitoring defaulting entrepreneurs.

IV.2. **Solving the model.** Because the economy exhibits a trend, we stationarize variables by their corresponding trend. We then rescale and linearize the model around the steady state equilibrium.[9] A detailed derivation of the steady-state equilibrium for the stationary variables is provided in the Online Appendix. The model is then solved using the solution algorithm based on the Mean Square Stable (MSS) concept proposed by Farmer, Waggoner, and Zha (2009), Farmer, Waggoner, and Zha (2011), and Cho (2016). Such an algorithm allows agents to take into account the possibility of future regime shifts when forming expectations. For efficiency and speed reasons, we use the Cho (2016)'s algorithm, which uses a forward method.

The solution can be characterized as follows

$$f_t = c(\chi_t, \theta, P) + T(\chi_t, \theta, P)f_{t-1} + R(\chi_t, \theta, P)\varepsilon_t, \tag{24}$$

where $c$ is the constant terms, $f_t$ is the vector of endogenous components, $\varepsilon_t$ is a vector of exogenous shocks, $\theta$ is a vector containing all structural parameters. As it can be seen, the law of motion of the model depends on the structural parameters ($\theta$), the prevailing regime ($\chi_t$), and the probability of switching across regimes ($P$).

IV.3. **Empirical Results.** This section provides the main quantitative results from the estimated MS-DSGE model. First, we present our estimation strategy in Section IV.3.1. Second, we report the estimates of structural parameters in Section IV.3.2 and we highlight

[9]As in Schorfheide (2005), Liu, Waggoner, and Zha (2011), and Bianchi, Ilut, and Schneider (2018), regime changes affect the steady state. To define the steady state equilibrium, we take the ergodic mean of each regime switching parameter, $\bar{x}$, as follows: $\bar{x} = \bar{p}_1 x(1) + \bar{p}_2 x(2)$, where $\bar{p}_i$ stands for the ergodic probability of being in regime $i$ and $x(i)$ is a parameter in regime $i$. We then refer to $\bar{y}$ as the value of the endogenous variable $y_t$ at the ergodic steady state given by $\bar{y} = f\left(\bar{x}\right)$, where $f\left(.\right)$ is the steady-state function which maps the values of endogenous variables to the ergodic values of structural parameters.

the key role of monitoring cost. Third, we present, in Section IV.3.3, the impulse response functions to uncertainty shock.

IV.3.1. *Estimation strategy.* Our estimation strategy is analogous to the impulse response matching approach used by Rotemberg and Woodford (1997) and Christiano, Eichenbaum, and Evans (2005), except that we are estimating the parameters to fit our regime-dependent impulse responses from a MS-SVAR, as opposed to impulse responses from a constant-parameters SVAR.[10] Our empirical analysis matches the estimated conditional impulse response functions of each endogenous variable obtained from the identified MS-SVAR model.

While output and credit spread[11] are directly observable from the theoretical model, this is not the case for our uncertainty measure. To the best of our knowledge, Basu and Bundick (2017) are the first to define the VIX index in a DSGE model, but this requires a third-order approximation to the model policy functions. At this stage, there is no efficient estimation algorithm to allow high-order approximations for MS-DSGE models. Nevertheless, it should be stressed that Foerster, Rubio-Ramírez, Waggoner, and Zha (2016) attempt to fill part of this gap using perturbation methods. However, their solution method is not enough fast and accurate to be used in an estimation algorithm.

That being said, Leahy and Whited (1996), Bloom, Bond, and Reenen (2007), and Bloom (2009) document that a number of cross-sectional measures of uncertainty are highly correlated with time-series stock-market volatility. In particular, Bloom (2009) presents evidence that the cross-sectional standard deviation of firm-level stock returns can be used as a proxy for time-series stock-market volatility since they are strongly correlated. Motivated by this evidence, we compute the analog of Bloom's cross-sectional uncertainty measure in our model and use it as a proxy for the VIX index. Following Christiano, Motto, and Rostagno (2014),

[10]We thank Mathias Trabandt for sharing computer codes used in Christiano, Trabandt, and Walentin (2010) on inference of constant DSGE models with the standard impulse response matching approach. We adapt their codes into Markov-switching environment.

[11]For the credit spread, we consider the Christiano, Motto, and Rostagno (2014)'s definition of the risk premium in the model which takes into account net expected losses in the case of default:

$$spread_t = Z_{t+1} - R_t - F_t(\bar{\omega}_{t+1})Z_{t+1} + \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega) R^k_{t+1} \frac{Q_{\bar{K},t}\bar{K}_{t+1}}{Q_{\bar{K},t}\bar{K}_{t+1} - N_{t+1}}.$$

we compute such a measure from the standard deviation, $std$, of the entrepreneurial return on equity in a cross-section including only nonbankrupt entrepreneurs (i.e., those with $\omega > \bar{\omega}$) as $std\left(R_t^e(\omega)|\omega > \bar{\omega}_t\right) = R_t^k L_{t-1}\sqrt{\mathbb{VAR}\left(\omega - \bar{\omega}_t|\omega > \bar{\omega}_t\right)}$, where $L_{t-1}$ denotes leverage, $R_t^k$ is the cross-sectional average return on capital, and $\mathbb{VAR}(x|D)$ denotes the variance of $x$ conditional on the event $D$.[12]

We link the two measures through the following measurement equation:

$$VIX_t = \kappa E_t\left\{std\left(R_{t+1}^e(\omega)|\omega \geq \bar{\omega}_{t+1}\right)\right\}, \tag{25}$$

where $\kappa$ is a constant of proportionality. Equation (25) determines the VIX index, $VIX_t$, as a function of the expectation of the standard deviation of the entrepreneurial return on equity.

As for the MS-SVAR model, we compute DSGE impulse responses by assuming that a particular regime is in place over the entire sample. Let $\tilde{\xi}$ is a $N \times 1$ vector, which stacks the contemporaneous and 16 lagged responses to each of three endogenous variables to the uncertainty shock. The number of elements in $\tilde{\xi}$ is equal to 2 (i.e., the number of regimes) times 3 (i.e., the number of variables) times 17 (i.e., the horizon) = 102 elements. Let $\xi(\theta)$ denotes the mapping from $\theta$ to the MS-DSGE model impulse response functions, with $\theta$ is a vector containing all estimated parameters. The likelihood function of the data, $\tilde{\xi}$ is defined as as function of $\theta$:

$$f(\tilde{\xi}|\theta,\bar{V}) = \left(\frac{1}{2\pi}\right)^{\frac{N}{2}} |\bar{V}^{-\frac{1}{2}}| \times \left[-\frac{1}{2}(\tilde{\xi} - \xi(\theta)')\bar{V}^{-1}(\tilde{\xi} - \xi(\theta))\right], \tag{26}$$

where $\bar{V}$ is a diagonal matrix with the sample variances of the $\tilde{\xi}$'s along the diagonal. Conditional on $\tilde{\xi}$ and $\bar{V}$, the Bayesian posterior of $\theta$ is as follows:

$$f\left(\theta, \bar{V}\right) \propto f(\tilde{\xi}|\theta,\bar{V}) \times f(\theta), \tag{27}$$

where $f(\theta)$ denotes the priors on $\theta$.

[12]Conditional on the period $t$ aggregate shocks, an entrepreneur with idiosyncratic shock $\omega$ earns $R_t = \max 0, [\omega - \bar{\omega}_t] \times R_t^k L_{t-1}$. Following Christiano, Motto, and Rostagno (2014), $\mathbb{VAR}\left(\omega - \bar{\omega}_t|\omega > \bar{\omega}_t\right)$ is given as follows

$$\mathbb{VAR}\left(\omega - \bar{\omega}_t|\omega > \bar{\omega}_t\right) = \frac{1}{1-F(\bar{\omega}_t)} e^{\sigma_t^2}\left[1 - \Phi\left(\frac{\log\bar{\omega}_t}{\sigma_t} - \frac{3}{2}\sigma_t\right)\right] - \left(\frac{1-G(\bar{\omega}_t)}{1-F(\bar{\omega}_t)}\right)^2,$$

with $\Phi(.)$ denotes the cumulative density function of a standard normal distribution.

The strategy of estimation begins by maximizing the logarithm of equation (27) using the `CSMINWEL` program, the optimization routine developed by Christopher A. Sims. Once at the posterior mode, we can start a Markov Chain Monte Carlo method to sample the posterior distribution. More specifically, we employ the Random-walk Metropolis Hasting procedure to generate draws from the joint posterior distribution of the MS-DSGE model. The results shown in the paper is based on 50,000 draws. We discard the first 10 percent draws as burn-in, and every 10th draw is retained.

IV.3.2. *Estimates of key parameters.* In order to keep the estimation procedure tractable, we calibrate several parameters. All of them are set along the lines of those estimated (at the mode) by Christiano, Motto, and Rostagno (2014). Table 3 summarizes it.

TABLE 3. Calibration of structural parameters.

| | | | | | |
|---|---|---|---|---|---|
| $\alpha$ | Capital share | 0.40 | $\pi^{target}$ | SS inflation rate | 2.43 |
| $\delta$ | Depreciation rate | 0.025 | $\sigma_c$ | elasticity utility | 1.5073 |
| $\lambda_w$ | SS markup, labor | 1.05 | $\rho_R$ | Taylor rule smoothing | 0.8519 |
| $\sigma_L$ | Curvature disutility of labor | 1.00 | $F(\bar{\omega})$ | SS probability of default | 0.0056 |
| $h$ | Consumption habit | 0.74 | $\lambda_w$ | SS markup, suppliers of labor | 1.05 |
| $\psi_L$ | Disutility weight on labor | 0.7705 | $W^e$ | Transfer received by entrepreneurs | 0.005 |
| $\beta$ | Discount factor | 0.9987 | $1-\gamma$ | Net worth transfer | $1-0.985$ |
| $\sigma_a$ | Elas. capital utilization costs | 2.54 | $\lambda_f$ | SS markup, intermediate good firms | 1.20 |
| $\tau^c$ | Tax rate consumption | 0.05 | $\eta_g$ | SS g/GDP ratio | 0.20 |
| $\tau^k$ | Tax rate capital | 0.32 | $\mu_z$ | Growth rate of the economy | 0.41 |
| $\tau^l$ | Tax rate labor | 0.24 | $\Upsilon$ | Trend rate of investment-specific technological change | 0.42 |

*Note:* Calibration is based on the calibrated and estimated parameters (at the mode) in Christiano, Motto, and Rostagno (2014).

Table 4 reports the specific distribution, the mean and the standard deviation for each estimated parameter. Most of the prior distributions for the parameters follow closely those in Christiano, Motto, and Rostagno (2014). Also, all priors remain unchanged across regimes, so that the data, through the likelihood, dominate the posterior distribution. Thus, our results are not be driven by asymmetric priors.

Table 4. Prior and posterior distribution.

| | | Prior | | | Posterior | | |
|---|---|---|---|---|---|---|---|
| Coefficient | Description | Density | Mean | Std Dev | Mode | [5; | 95] |
| $\xi_p(\chi_t = 1)$ | Calvo price | B | 0.50 | 0.10 | 0.4874 | 0.3113 | 0.6454 |
| $\xi_p(\chi_t = 2)$ | Calvo price | B | 0.50 | 0.10 | 0.4848 | 0.3231 | 0.6477 |
| $\xi_w(\chi_t = 1)$ | Calvo wage | B | 0.50 | 0.10 | 0.7926 | 0.5738 | 0.8974 |
| $\xi_w(\chi_t = 2)$ | Calvo wage | B | 0.50 | 0.10 | 0.7546 | 0.5458 | 0.8774 |
| $\iota_p(\chi_t = 1)$ | Indexation price | B | 0.50 | 0.15 | 0.5138 | 0.2493 | 0.7464 |
| $\iota_p(\chi_t = 2)$ | Indexation price | B | 0.50 | 0.15 | 0.4913 | 0.2453 | 0.7446 |
| $\iota_w(\chi_t = 1)$ | Indexation wage | B | 0.50 | 0.15 | 0.5218 | 0.2538 | 0.7538 |
| $\iota_w(\chi_t = 2)$ | Indexation wage | B | 0.50 | 0.15 | 0.4383 | 0.2236 | 0.7239 |
| $\phi_\pi(\chi_t = 1)$ | Taylor rule, inflation | N | 1.50 | 0.25 | 1.5073 | 1.1040 | 1.9050 |
| $\phi_\pi(\chi_t = 2)$ | Taylor rule, inflation | N | 1.50 | 0.25 | 1.5260 | 1.1464 | 1.9324 |
| $\phi_y(\chi_t = 1)$ | Taylor rule, output gap | G | 0.25 | 0.10 | 0.2464 | 0.0785 | 0.4045 |
| $\phi_y(\chi_t = 2)$ | Taylor rule, output gap | G | 0.25 | 0.10 | 0.2514 | 0.0907 | 0.4193 |
| $S''(\chi_t = 1)$ | Investment adjustment costs | G | 0.75 | 0.50 | 0.6145 | 0.2660 | 1.2983 |
| $S''(\chi_t = 2)$ | Investment adjustment costs | G | 0.75 | 0.50 | 1.1692 | 0.6192 | 1.8800 |
| $\mu(\chi_t = 1)$ | Monitoring cost | N | 0.275 | 0.10 | 0.2146 | 0.1229 | 0.3271 |
| $\mu(\chi_t = 2)$ | Monitoring cost | N | 0.275 | 0.10 | 0.0602 | 0.0399 | 0.1218 |
| $\rho_\sigma(\chi_t = 1)$ | Persistence shock | B | 0.60 | 0.20 | 0.5998 | 0.3253 | 0.6967 |
| $\rho_\sigma(\chi_t = 2)$ | Persistence shock | B | 0.60 | 0.20 | 0.7580 | 0.6913 | 0.8030 |
| $\sigma_\sigma(\chi_t = 1)$ | Std Dev shock | Inv-G | 1.00 | 1.00 | 0.3995 | 0.3145 | 0.5439 |
| $\sigma_\sigma(\chi_t = 1)$ | Std Dev shock | Inv-G | 1.00 | 1.00 | 0.4549 | 0.3258 | 0.5453 |
| $p_{11}$ | Transition matrix | D | 0.94 | 0.05 | 0.9847 | 0.8600 | 0.9926 |
| $p_{22}$ | Transition matrix | D | 0.94 | 0.05 | 0.9891 | 0.8823 | 0.9929 |
| $\kappa$ | Measurement VIX | N | 1.00 | 2.00 | 0.7888 | 0.5744 | 1.3495 |

*Note:* N stands for Normal, B Beta, D for Dirichlet, G for Gamma, and Inv-G for Inverted-Gamma. The 5 percent and 95 percent demarcate the bounds of the 90 percent probability interval.

First, we discuss the prior distributions for the parameters that determine the degree of nominal rigidities. The priors for the Calvo price and wage parameters ($\xi_p$ and $\xi_w$) follow a beta distribution, with the mean 0.50 and the standard deviation 0.10. This implies that the average duration of price and wage contracts is about two quarters. The priors for the price and wage indexation parameters ($\iota_p$ and $\iota_w$) follow a beta distribution, with the mean 0.50 and the standard deviation 0.15.

Second, regarding monetary policy parameters, the prior for the responses to inflation, $\phi_\pi$, follows a normal distribution with the mean 1.50 and the standard deviation 0.25, and the prior for the responses to output gap, $\phi_y$, has a gamma distribution with the mean 0.25 and the standard deviation 0.10.

Third, regarding financial contract parameters, the mean and the standard deviation of the prior distribution for the monitoring cost, $\mu$, is 0.275 and 0.10, respectively, thus covering parameter space suggested by Carlstrom and Fuerst (1997). The prior distributions of the uncertainty shock process is weakly informative. We use a beta distribution for the persistence of the shock with the mean 0.60 and standard deviation 0.20. Regarding its shock variance, we impose an inverted gamma distribution, with the mean and the standard deviation equal to 1.00.[13]

Finally, the priors on the transition matrix, $p_{ij}$, follow a dirichlet distribution, with the mean 0.94 and standard deviation 0.05, implying a prior duration of four years.

The group of estimated parameters is stacked as follows:

$$\theta = [\xi_p(k), \xi_w(k), \iota_p(k), \iota_w(k), S''(k), \phi_\pi(k), \phi_y(k), \mu(k), \rho_\sigma(k), \sigma_\sigma(k), p_{11}, p_{22}, \kappa], \tag{28}$$

with $k = \{1, 2\}$.

The last three columns of Table 4 report the posterior mode with the 90 percent probability interval for each structural parameter. Clearly, neither real and nominal frictions nor monetary policy parameters seem to account for the differences in the dynamics across regimes. Indeed, the estimates for $\phi_\pi(k)$ and $\phi_y(k)$, under both regimes, indicate that each posterior mode is closely similar to the mean of the respective prior, meaning that our impulse responses contain little information about the behavior of monetary policy. Regarding the parameters that determine the degree of nominal price rigidity, the estimates for $\xi_p$ and $\iota_p$ appear to be unidentified as well since posterior and prior distributions seem identical under each regime. By contrast, the estimates for wage parameters are well-identified but appear very similar with $\xi_w(1) = 0.79$ and $\xi_w(2) = 0.75$ for re-optimization parameters, and

[13]The inverted gamma distribution is as follows $p_{IG}(\sigma|\nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$, where $\nu$ and $s$ are hyperparameters.

$\iota_w(1) = 0.52$ and $\iota_w(2) = 0.43$ for indexation parameters. The estimates for $S''(k)$ are about 0.61 and 1.16 in the distress and the tranquil regime, respectively, slightly lower than those reported in the literature. Their 90 percent probability intervals overlap, meaning that there are no important differences across regimes.

By contrast, the role of financial frictions in explaining dynamics is very much in evidence. Indeed, the estimates for the monitoring cost, $\mu$, differ considerably between the two regimes. While it represents about 6.02% of the firm's value prior to bankruptcy in the tranquil regime, it turns out to be relatively higher in the distress regime with a value of 21.46%. The fact that the probability intervals do not overlap reinforces the evidence that the macroeconomic impact of uncertainty shocks depend crucially on the degree of financial frictions in the economy. The persistence of shocks, $\rho_\sigma$ turns to be lower in the distress regime than in the tranquil regime, and the estimated standard deviations for shocks are almost identical across regimes, with $\sigma_\sigma(1) = 0.39$ and $\sigma_\sigma(2) = 0.45$. Finally, the persistence of each regime is fairly similar, with $p_{11} = 0.9847$ and $p_{22} = 0.9891$.

IV.3.3. *Impulse responses.* Figure 5 reports, in black line, the impulse responses of endogenous variables to the uncertainty shock obtained from the MS-DSGE model. Each column represents the responses of a particular variable under each regime. For comparison purposes, we also present the median and the 68 and 90 percent probability intervals of the MS-SVAR model-implied responses. A number of results are worth emphasizing here. First, the model performs well at accounting for the dynamic responses of the economy to a uncertainty shock. All the DSGE model-implied responses lie within the 68 percent probability intervals computed from the MS-SVAR model. From a qualitative point of view, the responses of output and credit spread in the tranquil regime share some common features with the responses in the distress regime. Credit spread and output move in opposite directions; output declines progressively, while credit spread rises immediately and then begins to return its pre-shock level steadily.

The transmission mechanism is straightforward. The uncertainty shock directly alters the degree of risk associated with the asymmetric information between lenders and entrepreneurs who borrow external funds to produce physical capital goods. It moves the dispersion of entrepreneurs' idiosyncratic productivity. With imperfect financial markets, this shock implies higher external finance costs since more entrepreneurs draw low levels of productivity and

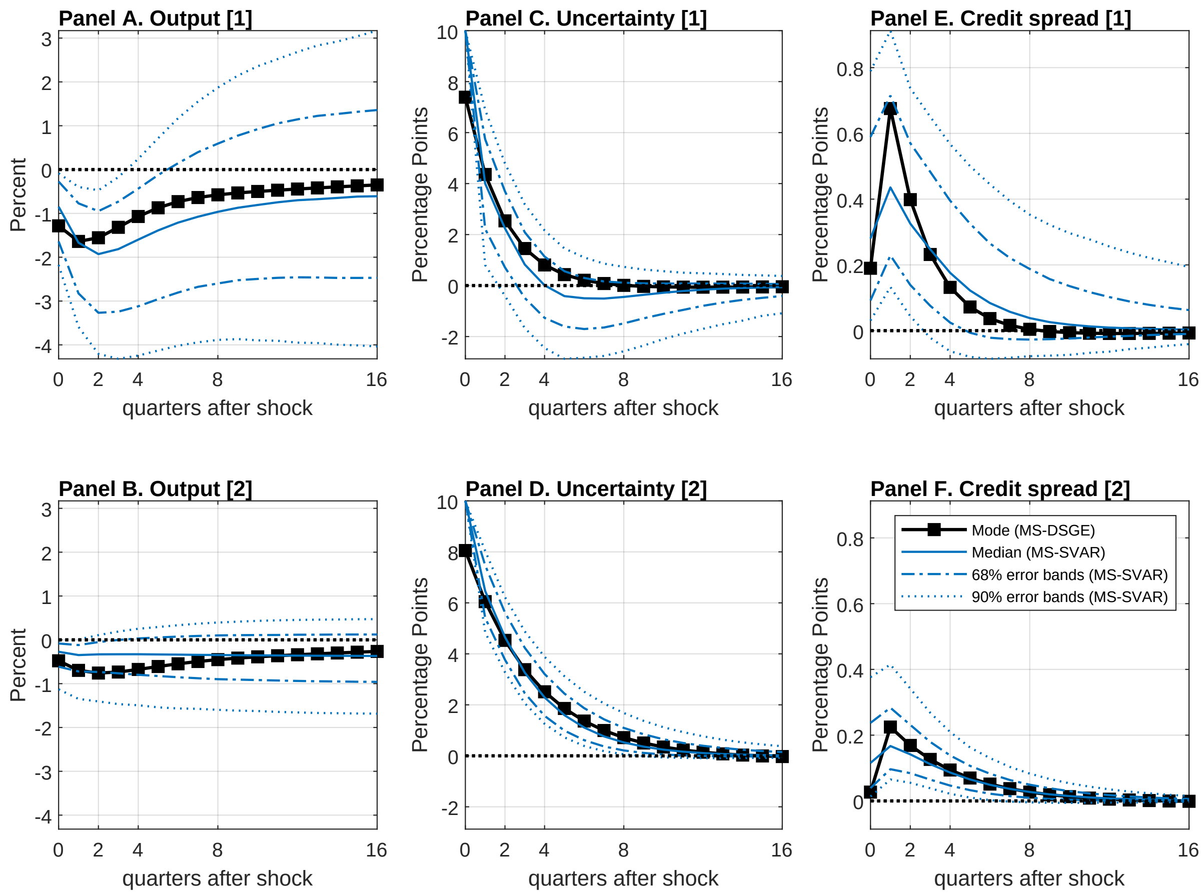


Figure 5. Impulse-response functions to a uncertainty shock. The median responses from the identified MS-SVAR model are reported in solid blue lines, and the 68% and 90% error bands in dotted blue lines. The black lines report the responses (at the mode) from the MS-DSGE model.

are then unable to reimburse their debts. Then, a positive uncertainty shock increases both the risk of default and the cost of external funds which lead to a fall in the economic activity of entrepreneurs transmitted to the overall economy in general equilibrium through an increase of the credit spread and a fall in investment and production. Say it differently, financial frictions act as the main mechanism through which changes in uncertainty affect macroeconomic variables.

Furthermore, the model succeeds in accounting for the differences in the responses of endogenous variables across the two regimes. Indeed, there is a notable change in the way both output and credit spread respond to the shock. Concerning the changes in the impulse responses between the two regimes, the responses under the distress regime are remarkably amplified compared to those in the tranquil regime. Under these circumstances, financial frictions act as an amplification mechanism.

This stronger effect of uncertainty in distress periods can be explained as follows. Because of the greater cost of information asymmetry in periods of distress, financial intermediaries charge higher premiums, as compared to normal periods. In our baseline model, this manifests itself in a greater sensitivity of premium to the firm's net worth in distress periods. In this context, a uncertainty shock causes larger credit spread increases, and therefore, larger and long-lasting negative effects in economic activity. In contrast, when stress is low, the economy is better capable of absorbing the coming economic shocks. As a result, the macroeconomic effects are less pronounced.

IV.4. **Further discussion on $\mu$.** The remarkable feature of our evidence is a strengthening asymmetry of information between borrowers and lenders in periods of distress, which manifests itself by higher monitoring costs. Our evidence is closely related to the estimates of monitoring cost in Lindé, Smets, and Wouters (2016). The authors estimate, with full information methods, a DSGE model with a Bernanke-Gertler-Gilchrist financial accelerator mechanism in which the monitoring cost is allowed to change according to a Markov-switching process. Interestingly, they capture changes in the degree of the financial frictions, with repeated changes in the monitoring costs between a low (2.90%) and high (8.40%) value over time. These estimated values appear to be lower than ours. This difference can be explained by two main reasons. First, they estimate a MS-DSGE with full information methods — i.e., key macroeconomic and financial variables are directly observable in the model — while we estimate our MS-DSGE by the impulse-response matching approach. Second, our MS-SVAR model takes properly into account heteroskedasticity of U.S. macroeconomic disturbances, while they do not. Indeed, Sims (2001), and more recently Lhuissier and Zabelina (2015), have shown the importance of capturing heteroskedasticity before allowing changes in economic dynamics in order to avoid misleading results. In Lindé, Smets, and Wouters (2016), only the monitoring cost parameter is allowed to change over time while shock variances remain constant. Our paper overcomes this issue by allowing both equation coefficients and shock variances to change over time independently.

Our results also corroborate with the existing literature that introduces time-varying monitoring costs into partial or general equilibrium models. For example, Levin, Natalucci, and Zakrajšek (2004) and Fuentes-Albero (2019) show the estimate for $\mu$ varies between 0% and 50% over the period 1954.Q4 to 2006.Q4, with peaks in periods of financial distress. This wide range covers the estimates for our monitoring cost, $\mu(\chi_t)$, in both regimes.

Our evidence of regime-dependent monitoring costs appears to be consistent with financial literature that reports higher bankrupt costs in periods of distress. Frye (2000a,b) pleads for consideration of the *double misfortune* in credit risk model: during crises “many obligators default, and the value of collateral is damaged”. Indeed, as emphasized by Altman, Brady, Resti, and Sironi (2005) there is a strong negative correlation between the recovery rate and the rate of default. Acharya, Bharath, and Srinivasan (2007) supplement this evidence by showing that the recovery rate of creditors from default or bankrupt is $10\% - 15\%$ lower in distressed industries than in healthy industries.

The common explanation to these empirical facts, and by consequence rational for time-varying monitoring costs in DSGE models, tracks back to Shleifer and Vishny (1992) model of fire sales which explain that assets are sold at a discount price during periods of financial stress. Discounted prices of assets imply higher costs of bankrupt in distress periods, e.g. higher $\mu$ as assumed in our model. Interestingly, recently Candian and Dmitriev (2019) include the fire sales mechanism in the Bernanke, Gertler, and Gilchrist (1999) setup assuming that the monitoring cost parameter $\mu$ is a function of market liquidity defined the ratio of net sales over net purchases of capital in the capital good markets.

IV.5. **Alternative specifications.** In our baseline model with regime changes in several structural parameters, posterior and prior distributions appear to be identical for some of them. This might be due to the limited number of impulse responses that we use. It would be useful to consider some alternative specications in which those parameters are fixed along the line suggested by previous studies. For this purpose, we re-estimate several restricted versions of the baseline model, shutting down or calibrating one category of parameters at-a-time, and study the resulting posterior distributions.

The restricted models under consideration are the following: first and second, models with constant Taylor rule parameters fixed at those estimated in Christiano, Motto, and Rostagno (2014) and Liu, Waggoner, and Zha (2011), which corresponds to $\phi_\pi = 2.40$ and

$\phi_y$=0.36 and $\phi_\pi = 1.655$ and $\phi_y = 0.006$, respectively;[14] third, a model in which nominal rigidities (Calvo prices and wages, partial indexation of prices and wages to past inflation) are calibrated with those estimated in Christiano, Motto, and Rostagno (2014), i.e., $\xi_p = 0.74$, $\xi_w = 0.81$, $\iota_p = 0.90$, and $\iota_w = 0.49$; and fourth, a model with flexible prices and wages, which corresponds to $\xi_p = \xi_w = 0.00$.

The results of this exercise are reported in the Online Appendix. The main result to note is that variations in specification do not alter the findings reported by the baseline model. There are still large differences across regimes in the monitoring cost, whose the magnitudes of estimates are remarkably similar to that reported in the baseline specification.

Finally, the model with flexible prices and wages also emphasizes changes in the systematic behavior of monetary policy. The mode estimate for $\phi_\pi(\chi_t)$ is 1.51 in Regime 1 and 2.06 in Regime 2, meaning that monetary policy is less aggressive in distress periods. However, one cannot conclude that monetary policy plays a role in the changing transmission mechanism of uncertainty shocks in the flexible prices and wages model for two main reasons. First, 90% error bands overlap, thus indicating that monetary policy changes are not very much in evidence. Second, the model's fit is far below to our baseline model, as shown by the marginal data densities (or marginal likelihoods). The marginal likelihood of the baseline model is about 22 log-points higher than that of the flexible prices and wages model, thus implying overwhelming posterior odds in its favor.

IV.6. **Expectation effects of regime shifts in financial conditions.** In the previous section, we have illustrated the role of financial frictions in propagating uncertainty shocks by comparing economic outcomes of two possible regimes: one regime with a high monitoring cost, and another regime with a low monitoring cost. Results were not only based on the estimated structural parameters of each regime, but also on the transition matrix used by agents when forming their expectations. In this section, we gauge what would have happened if agents had considered different probabilities of moving across regimes. Such a counterfactual is interesting to execute because it allows assessing the role of expectation effects of regime switching in financial conditions.

[14]Because it is common to observe large differences in the estimates of Taylor rule parameters across different studies, we use two distinct estimates. The choice of using the Liu, Waggoner, and Zha (2011)'s estimates appear to be relevant since their DSGE model allows for regime changes.

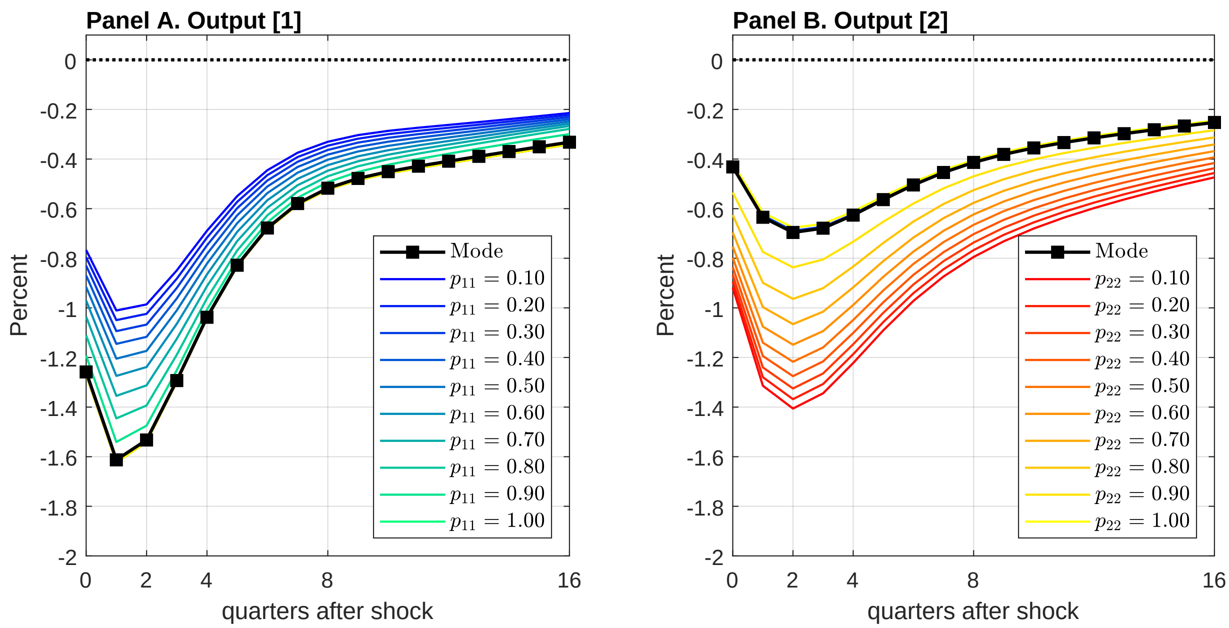


FIGURE 6. Impulse-response functions to uncertainty shock as a function of the probability of staying in the distress regime (panel A), and in the tranquil regime (panel B).

Figure 6 displays the conditional impulse responses of output following a uncertainty shock when agents' beliefs about the probability of staying in the same regime varies between 0.00 to 1.00. Our uncertainty and credit spread variables are not directly concerned by the expectation effect. When considering $p_{ii} = 1$, agents believe that the regime in which they are will last indefinitely. Inversely, the more $p_{ii}$ declines, the more agents believe that the economy will move to the other regime in the next period.

Clearly, the expectation effects play an important role in shaping the dynamic behavior of economic activity. As one can see, if agents take into account the effects of possible changes in future financial conditions, macroeconomic outcomes are remarkably altered. The more are agents optimistic about future financial conditions (i.e., gradual moves toward $p_{11} = 0$ in the left column or $p_{22} = 1$ in the right column), the more macroeconomic effects are dampened. Reciprocally, the pessimism of agents about financial conditions (i.e., gradual moves toward $p_{11} = 1$ in the left column or $p_{22} = 0$ in the right column) amplifies the effects of uncertainty shocks. Quantitatively, the expectation effects appear to be bigger under the tranquil regime, where the output effects of the shock can be divided by two. The role of expectation effects of regime switching in the degree of financial frictions appears to be important in amplifying or mitigating the propagation of uncertainty shocks. Therefore, these expectation effects are an important component of the financial accelerator mechanism.

## V. Conclusion

Why are the real effects of uncertainty shocks so different over time? Our results point to a key role for changes in the degree of financial frictions; the financial accelerator is strengthened in distress periods. Under these circumstances, agents' expectations around the level of frictions can alter macroeconomic outcomes. Optimistic expectations about future financial conditions dampen contractionary effects of uncertainty shocks on aggregate activity. Conversely, pessimistic expectations amplify their effects.

These conclusions have important implications for the conduct of monetary and macro-prudential policies. For example, the bulk of the evidence suggests that these policies can reduce the frequency and severity of financial disruptions, and thus the likelihood of observing a regime characterized by a high degree of financial frictions. In this context, if policymakers communicate to and persuade, in a clear way, agents that such policies are around the corner, then they can, even before implementing them, dampen the adverse effects of uncertainty shocks. The ability of policymakers to manage agents' expectations reveals to be crucial in shaping business cycle fluctuations.

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# ONLINE APPENDIX: REGIME-DEPENDENT EFFECTS OF UNCERTAINTY SHOCKS: A STRUCTURAL INTERPRETATION

*Not for Publication*

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This Appendix consists of the following sections:

A. Data
B. Bayesian inference for MS-SVAR model
C. Robustness results for MS-SVAR model
D. Equilibrium conditions of MS-DSGE model
E. Results for alternative specifications of MS-DSGE model

## Appendix A. Data

All data are organized quarterly from the second Quarter of 1962 to the second Quarter of 2018. Most data comes from Federal Reserve Economic Database (FRED).

- $gdp_t$: output is the real GDP (GDPC1).
- $vix_t$: uncertainty is the Chicago Board of Options Exchange Market Volatility Index. From 1963 to 2009, we use the constructed index by Bloom (2009). Then, from 2009, we follow Stock and Watson (2012) and take a quarterly average of daily VIX.
- $sp_t$: credit spread is constructed as the difference between BAA corporate bond yields (BAA) and AAA corporate bond yields (AAA).

For inference, we use the natural log of output. Our spread and uncertainty variables remain unchanged.

## Appendix B. Bayesian inference for MS-SVAR model

This section provides a detailed description of the Bayesian inference employed in this paper. More specifically, we closely follow Sims, Waggoner, and Zha (2008).

B.1. **The posterior.** Before describing the posterior distribution, we introduce the following notation: $\theta$ and $q$ are vectors of parameters where $\theta$ contains all the parameters of the model (except those of the transition matrix) and $q = (q_{i,j}) \in \mathbb{R}^{h^2}$. $Y_t = (y_1, \ldots, y_t) \in (\mathbb{R}^n)^t$ are observed data with $n$ denoting the number of endogenous variables and $S_t = (s_0, \ldots, s_t) \in H^{t+1}$ with $H \in \{1, \ldots, h\}$.

The log-likelihood function, $p(Y_T|\theta, q)$, is combined with the prior density functions, $p(\theta, q)$, to obtain the posterior density, $p(\theta, q|Y_T) = p(\theta, q)p(Y_T|\theta, q)$.

B.1.1. *The likelihood.* Following Hamilton (1989), Sims and Zha (2006), and Sims, Waggoner, and Zha (2008), we employ a class of Markov-switching structural VAR models of the following form:

$$y_t' A(s_t) = x_t' F(s_t) + \varepsilon_t' \Xi^{-1}(s_t), \tag{B.1}$$

with $x_t' = \begin{bmatrix} y_{t-1}' & \cdots & y_{t-\rho}' & 1 \end{bmatrix}$ and $F(s_t) = \begin{bmatrix} A_1(s_t) & \cdots & A_\rho(s_t) & C(s_t) \end{bmatrix}'$.

Let $a_j(k)$ be the $j$th column of $A(k)$, $f_j(k)$ be the $j$th column of $F(k)$, and $\xi_j(k)$ be the $j$th diagonal element of $\Xi(k)$. The conditional likelihood function is as follows:

$$p(y_t|s_t, Y_{t-1}) = |A(s_t)| \prod_{j=1}^{n} |\xi_j(s_t)| \exp\left(-\frac{\xi^2(s_t)}{2}\left(y_t' a_j(s_t) - x_t' f_j(s_t)\right)^2\right). \tag{B.2}$$

To simplify the Gibbs-sampling procedure described in the next section, it is preferable to rewrite the condition likelihood function with respect to free parameters from matrix $A(s_t)$ and $F(s_t)$:

$$|A(s_t)| \prod_{j=1}^{n} |\xi_j(s_t)| \exp\left(-\frac{\xi^2(s_t)}{2}\left((y_t' + x_t' W_j) U_j b_j(s_t) - x_t' V_j g_j(s_t)\right)^2\right), \tag{B.3}$$

where $a_j(s_t) = U_j b_j(k)$ and $f_j(s_t) = V_j g_j - W_j U_j b_j(k)$ is a result from the linear restrictions $R_j \begin{bmatrix} a_j & f_j \end{bmatrix}' = 0$; and $U_j$ and $V_j$ are matrices with orthonormal columns and $W_j$ is a matrix. See Waggoner and Zha (2003) for further details.

The log likelihood function is given by

$$p(Y_T|\theta, q) = \sum_{t}^{T} \ln\left\{\sum_{s_t=1}^{h} p(y_t|s_t, Y_{t-1}) \Pr\left[s_t|Y_{t-1}\right]\right\}, \tag{B.4}$$

where

$$\Pr\left[s_t = i|Y_{t-1}\right] = \sum_{j=1}^{h} \Pr\left[s_t = i, s_{t-1} = j|Y_{t-1}\right] \tag{B.5}$$

$$= \sum_{j=1}^{h} \Pr\left[s_t = i|s_{t-1} = j\right] \Pr\left[s_{t-1} = j|Y_{t-1}\right]. \tag{B.6}$$

with $q_{i,j} = \Pr\left[s_t = i|s_{t-1} = j\right]$ are the transition probabilities from the $h \times h$ matrix $Q$

$$Q = \begin{bmatrix} q_{1,1} & \cdots & q_{1,j} \\ \vdots & \ddots & \vdots \\ q_{i,1} & \cdots & q_{i,j} \end{bmatrix} \tag{B.7}$$

The probability terms are updated as follows:

$$\Pr\left[s_t = j|Y_t\right] = \Pr\left[s_t = j|Y_{t-1}, y_t\right] = \frac{p(s_t = j, y_t|Y_{t-1})}{p(y_t|Y_{t-1})} \tag{B.8}$$

$$= \frac{p(y_t|s_t = j, Y_{t-1}) \Pr[s_t = j|Y_{t-1}]}{\sum_{j=1}^{h} p(y_t|s_t = j, Y_{t-1}) \Pr[s_t = j|Y_{t-1}]}. \tag{B.9}$$

B.1.2. *The prior.* Following Sims and Zha (1998), we exploit the idea of a Litterman's random-walk prior from structural-form parameters. Note that dummy observations are not introduced as a component of the prior to keep in line with the original Litterman's prior. Using linear restrictions, the overall prior, $p(\theta, q)$, is given in the following way:

$$p(b_j(k)) = \text{normal}(b_j(k)|0, \bar{\Sigma}_{b_j}), \tag{B.10}$$

$$p(g_j(k)) = \text{normal}(g_j(k)|0, \bar{\Sigma}_{g_j}), \tag{B.11}$$

$$p(\xi_j^2(k)) = \text{gamma}(\xi_j^2(k)|\bar{\alpha}_j, \bar{\beta}_j), \tag{B.12}$$

$$p(q_j) = \text{dirichlet}(q_{i,j}|\alpha_{1,j}, \ldots, \alpha_{k,j}), \tag{B.13}$$

where $\bar{\Sigma}_{b_j}$, $\bar{\Sigma}_{\psi_j}$, and $\bar{\Sigma}_{\delta_j}$ denotes the prior covariance matrices and $\bar{\alpha}_j$ and $\bar{\beta}_j$ are set to one, allowing the standard deviations of shocks to have large values for some regimes.

The Gamma distribution is defined as follows:

$$\text{gamma}(x|\alpha, \beta) = \frac{1}{\Gamma(\alpha)} \beta^{\alpha} x^{\alpha-1} e^{-\beta x}. \tag{B.14}$$

Regarding the transition matrix, $Q$, suppose that $q_j = [q_{1,j}, \ldots, q_{h,j}]'$. The prior, denoted $p(q_j)$, follows a Dirichlet form as follows:

$$p(q_j) = \left( \frac{\Gamma\left(\sum_{i \in H} \alpha_{i,j}\right)}{\prod_{i \in H} \Gamma(\alpha_{i,j})} \right) \times \prod_{i \in H} (q_{i,j})^{\alpha_{i,j}-1}, \tag{B.15}$$

where $\Gamma$ denotes the standard gamma function.

B.2. **Gibbs-sampling.** Following Kim and Nelson (1999) and Sims, Waggoner, and Zha (2008), a Markov Chain Monte Carlo (MCMC) simulation method is employed to approximate the joint posterior density, $p(\theta, q, S_T|Y_T)$. The advantage of using VARs is that conditional distributions like $p(S_T|Y_T, \theta, q)$, $p(q|Y_T, S_T, \theta)$, and $p(\theta|Y_T, q, S_T)$ can be obtained in order to exploit the idea of Gibbs-sampling by sampling alternatively from these conditional posterior distributions.

B.2.1. *Conditional posterior densities,* $p(\theta|Y_T, q, S_T)$. To simulate draws of $\theta \in \{b_j(k), g_j(k), \xi_j^2\}$ from $p(\theta|Y_T, S_t, q)$, one can start to sample from the conditional posterior

$$p(b_j(k)|y_t, S_t, b_i(k)) = \exp\left(-\frac{1}{2}b_j'(k)\bar{\Sigma}_{b_j}^{-1}b_j(k)\right) \times \prod_{t\in\{t:s_t=k\}}\left[|A(k)|\exp\left(-\frac{\xi^2(s_t)}{2}(y_t'a_j(k) - x_t'f_j(k))^2\right)\right], \quad \text{(B.16)}$$

using the Metropolis-Hastings (MH) algorithm. Then a multivariate normal distribution is employed to draw $g_j(k)$:

$$p(g_j(k)|y_t, S_t) = \text{normal}(g_j(k)|\tilde{\mu}_{g_j(k)}, \tilde{\Sigma}_{g_j(k)}). \quad \text{(B.17)}$$

The computational details of the posterior mean vectors and covariance matrices are given in Sims, Waggoner, and Zha (2008).

Disturbance variances $\xi_j^2$ are simulated from a gamma distribution

$$p(\xi_j^2(k)|y_t, S_t) = \text{gamma}(\xi_j^2(k)|\tilde{\alpha}_j(k), \tilde{\beta}_j(k)), \quad \text{(B.18)}$$

where $\tilde{\alpha}_j(k) = \bar{\alpha}_j + \frac{T_{2,k}}{2}$ and

$$\tilde{\beta}_j(k) = \bar{\beta}_j + \frac{1}{2}\sum_{t\in\{t:s_{2t}=k\}}(y_t'a_j(s_t) - x_t'f_j(s_t))^2, \quad \text{(B.19)}$$

with $T_{2,k}$ denoting the number of elements in $\{t : s_{2t} = k\}$.

B.2.2. *Conditional posterior densities,* $p(S_T|Y_T, \theta, q)$. A multi-move Gibbs-sampling is employed to simulate $S_t, t = 1, 2, ..., T$. First, draw $s_t$ according to

$$p(s_t|y_t, S_t) = \sum_{s_{t+1}\in H} p(s_t|Y_T, \theta, q, s_{t+1})p(s_{t+1}|Y_T, \theta, q), \quad \text{(B.20)}$$

where

$$p(s_t|Y_t, \theta, q, s_{t+1}) = \frac{q_{s_{t+1},s_t}p(s_t|Y_t, \theta, q)}{p(s_{t+1}|Y_t, \theta, q)}. \quad \text{(B.21)}$$

Then, in order to generate $s_t$, one can use a uniform distribution between 0 and 1. If the generated number is less than or equal to the calculated value of $p(s_t|y_t, S_t)$, we set $s_t = 1$. Otherwise, $s_t$ is set equal to 0.

B.2.3. *Conditional posterior densities, $p(q|Y_T, S_T, \theta)$.* The conditional posterior distribution of $q_j$ is as follows:

$$p(q_j|Y_t, S_t) = \prod_{i=1}^{h} (q_{i,j})^{n_{i,j}+\beta_{i,j}-1}, \tag{B.22}$$

where $n_{i,j}$ is the number of transitions from $s_{t-1} = j$ to $s_t = i$.

## Appendix C. Robustness results for MS-SVAR model

### C.1. **The threshold of minimum contribution.** 60%, 70%, and 80%.

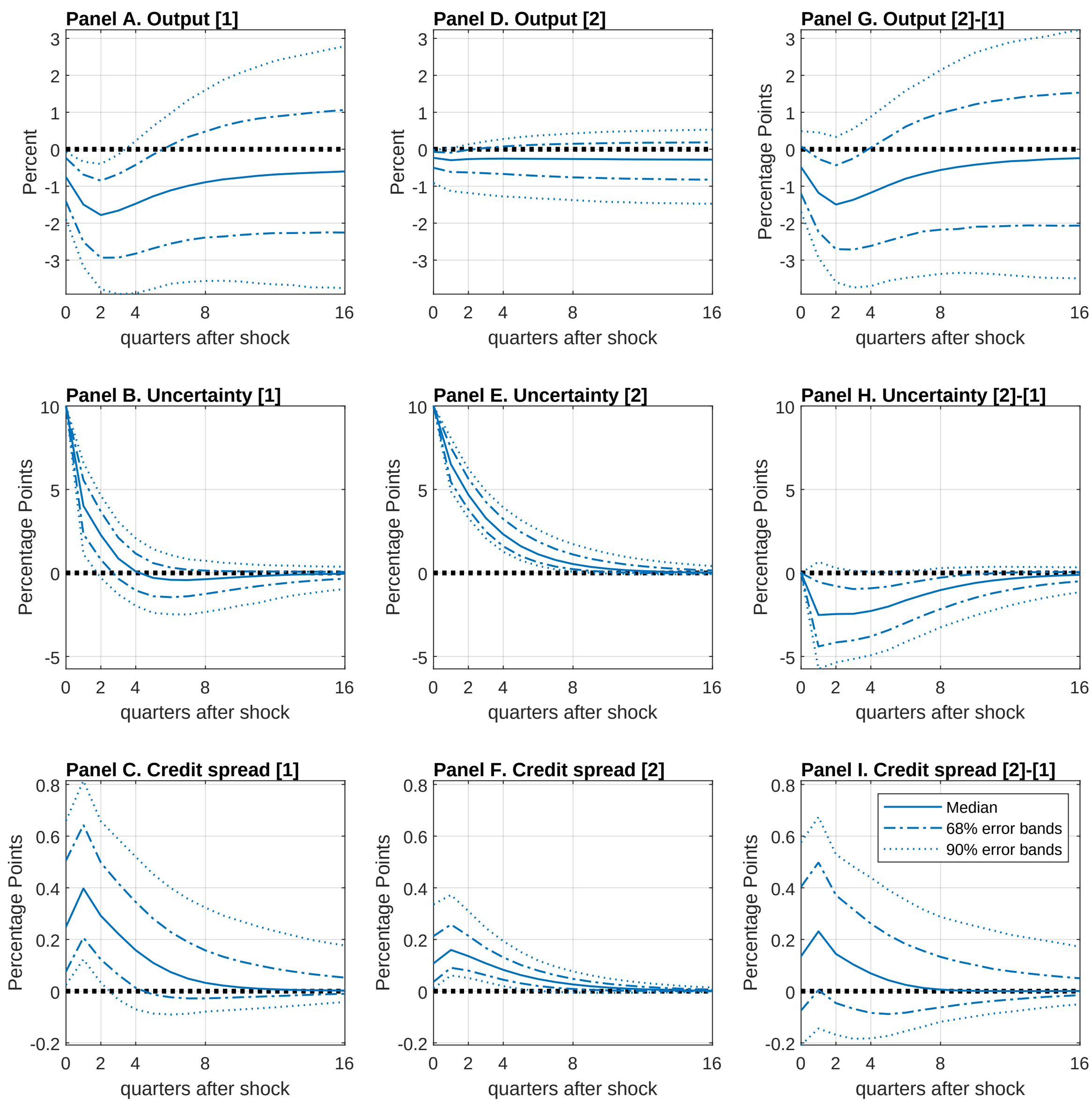


Figure 7. Impulse-response functions to uncertainty shock. Threshold of 60%.

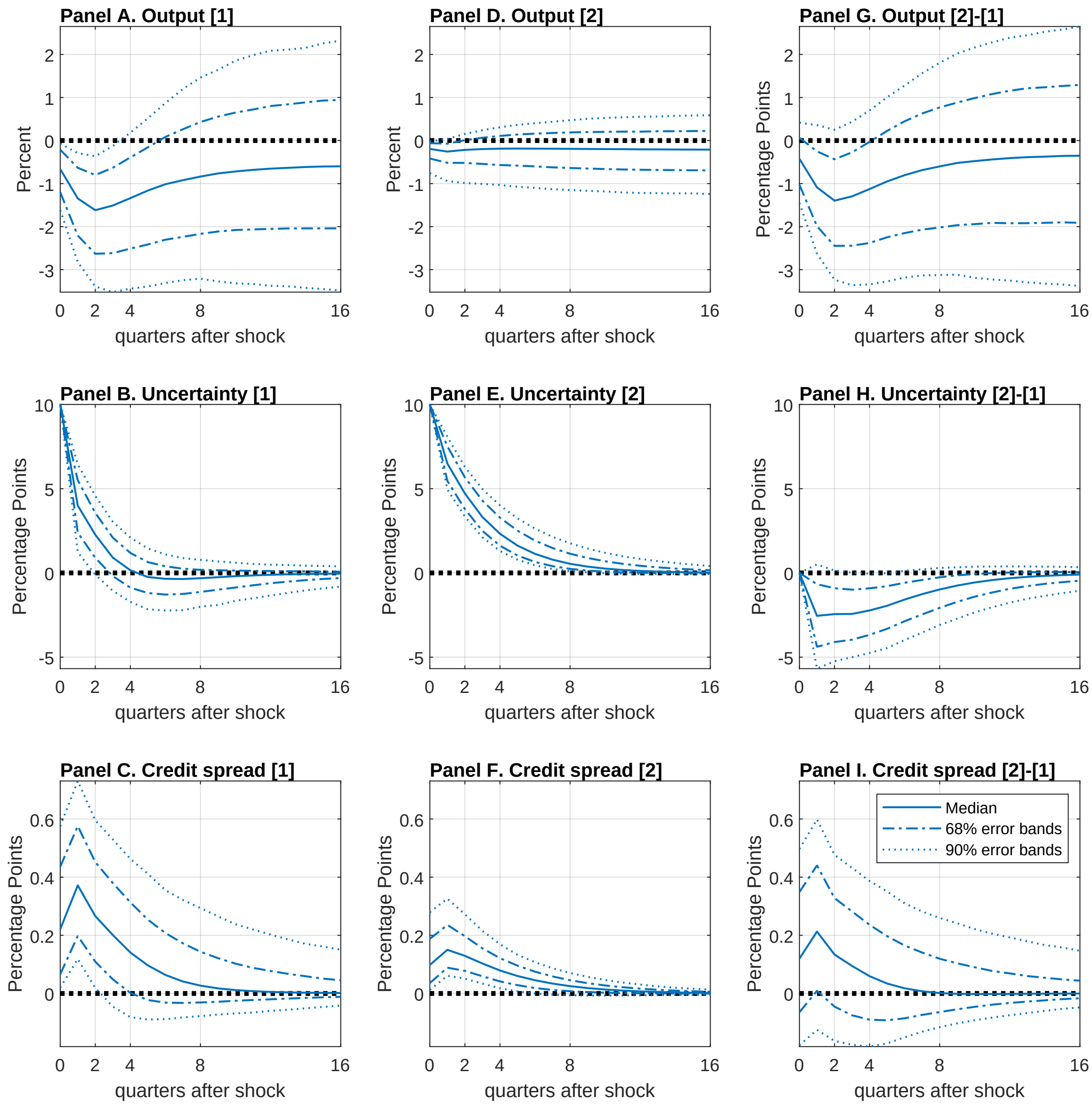


FIGURE 8. Impulse-response functions to uncertainty shock. Threshold of 70%.

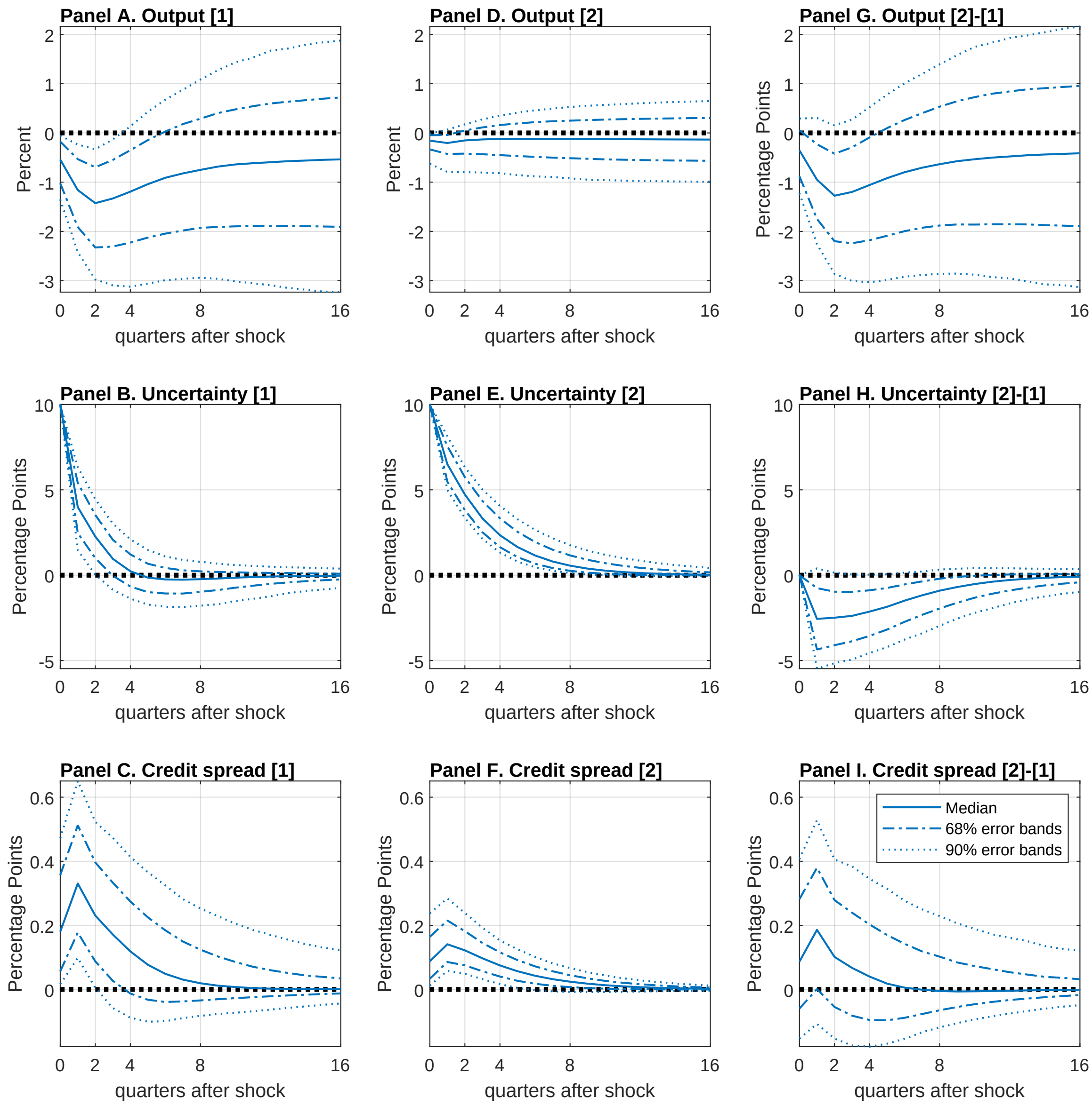


FIGURE 9. Impulse-response functions to uncertainty shock. Threshold of 80%.

C.2. **Alternative restrictions horizon.** Restrictions imposed for the next two quarters.

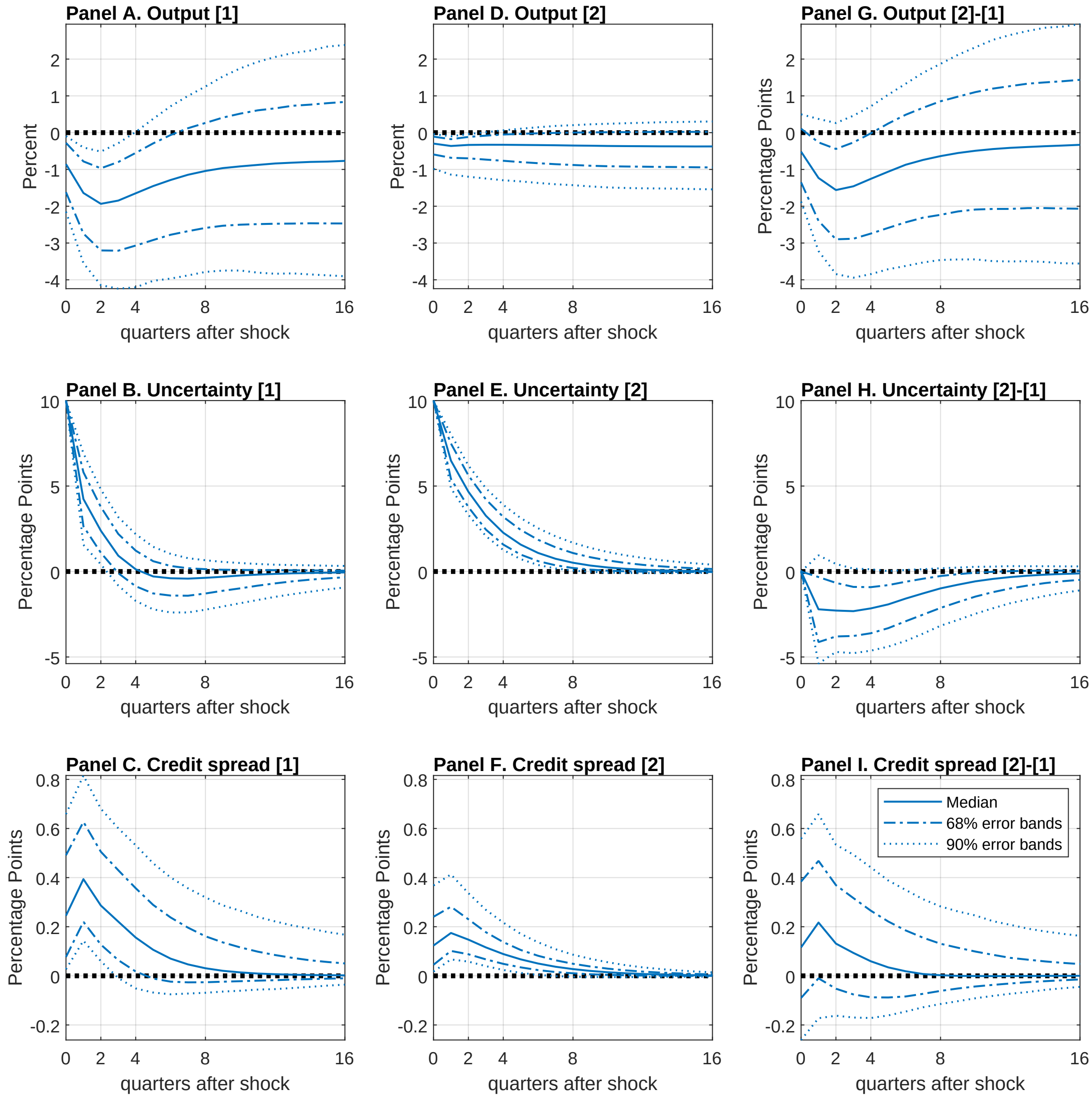


FIGURE 10. Impulse-response functions to uncertainty shock.

C.3. **Alternative restriction on forecast error variance.** Identification strategy in the spirit of Uhlig (2003).

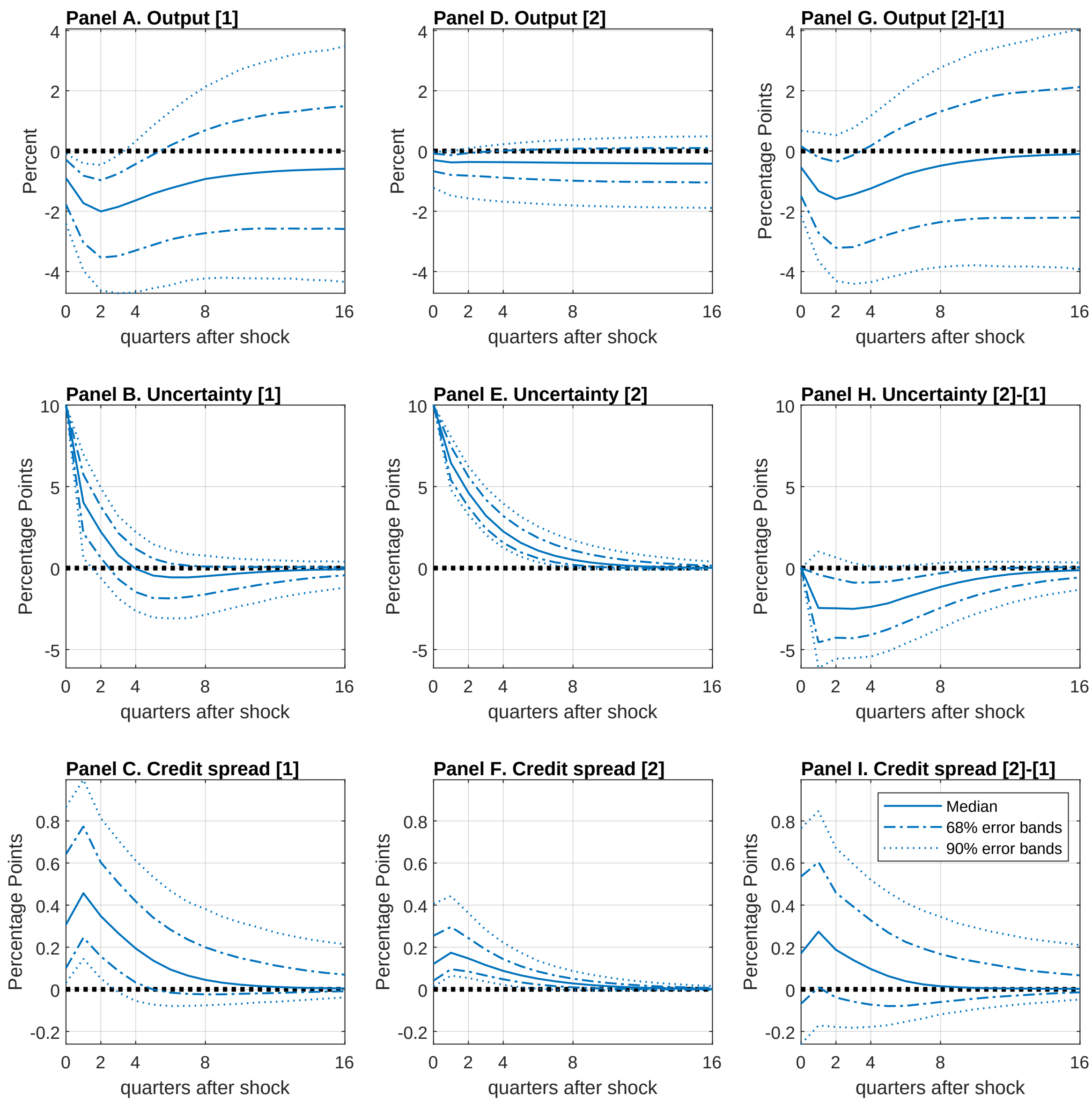


FIGURE 11. Impulse-response functions to uncertainty shock.

## Appendix D. Equilibrium Conditions of MS-DSGE model

To solve our model, we require that the variables be stationary. The level of neutral and investment-specific technology have a unit root. The composite trend is then $z_t^* = z_t \Upsilon^{(\frac{\alpha}{1-\alpha})t}$ with the following steady state growth rate:

$$z^* = z\Upsilon^{(\frac{\alpha}{1-\alpha})}. \tag{D.1}$$

Several variables are then transformed to induce stationarity as follows:

$$c_t = \frac{C_t}{z_t^*}, \qquad y_t = \frac{Y_t}{z_t^*}, \qquad i_t = \frac{I_t}{z_t^* \Upsilon^t}, \qquad k_t = \frac{K_t}{z_{t-1}^* \Upsilon^{t-1}}, \qquad n_t = \frac{N_t}{P_{t-1} z_{t-1}^*} \tag{D.2}$$

$$g_t = \frac{G_t}{z_t^*}, \qquad w_t = \frac{W_t}{z_t^* P_t}, \qquad \mu_{z,t}^* = \frac{z_t*}{z_{t-1}^*}, \qquad gdp_t = \frac{GDP_t}{z_t^*}, \qquad w_t^e = \frac{W_t^e}{P_t z_t^*}. \tag{D.3}$$

The nominal rental rate on capital $(\tilde{r}_t^k P_t)$ and the market price of capital are transformed to induce stationarity as well

$$r_t^k = \Upsilon^t \tilde{r}_t^k, \qquad q_t = \Upsilon^t \frac{Q_{K,t}}{P_t}. \tag{D.4}$$

We now re-write the model in a stationary form. The pricing equation by monopolistic producers is

$$p_t^* = \left( (1 - \xi_p(\chi_t)) \left( \frac{K_{p,t}}{F_{p,t}} \right)^{\frac{\lambda_f}{1-\lambda_f}} + \xi_p(\chi_t) \left( \frac{\tilde{\pi}_t}{\pi_t} p_{t-1}^* \right)^{\frac{\lambda_f}{1-\lambda_f}} \right)^{\frac{1-\lambda_f}{\lambda_f}} \tag{D.5}$$

with

$$F_{p,t} = \lambda_{z,t} y_t + \beta \mathrm{E}_t \left\{ F_{p,t+1}\, \xi_p(\chi_t) \left( \frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}} \right\}, \quad \text{and} \tag{D.6}$$

$$K_{p,t} = y_t \lambda_{z,t} \lambda_f s_t + \beta \mathrm{E}_t \left\{ \xi_p(\chi_t) \left( \frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_f}{1-\lambda_f}} K_{p,t+1} \right\}, \tag{D.7}$$

which satisfy

$$K_{p,t} = F_{p,t} \left( \frac{1 - \xi_p(\chi_t) \left( \frac{\tilde{\pi}_t}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1 - \xi_p(\chi_t)} \right)^{1-\lambda_f} \tag{D.8}$$

The inflation indexation rule is

$$\tilde{\pi}_t = \pi^{*\iota(\chi_t)}\, \pi_{t-1}^{1-\iota(\chi_t)} \tag{D.9}$$

The wage equation setting by labor contractor is

$$w_t^* = \left( (1-\xi_w(\chi_t)) \left( \frac{1-\xi_w(\chi_t)\left(\mu_z \frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w(\chi_t)} \right)^{\lambda_w} + \xi_w(\chi_t)\left(\mu_z \frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} w_{t-1}^*\right)^{\frac{\lambda_w}{1-\lambda_w}} \right)^{\frac{1}{\frac{\lambda_w}{1-\lambda_w}}} \tag{D.10}$$

with

$$F_{w,t} = \frac{h_t (w_t^*)^{\frac{\lambda_w}{\lambda_w-1}} \lambda_{z,t}(1-\tau_l)}{\lambda_w} + \beta \mathrm{E}_t \left\{ F_{w,t+1} \frac{\xi_w(\chi_t)\, \mu_z^{*\frac{\lambda_w}{1-\lambda_w}} \tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}} \left(\frac{1}{\tilde{\pi}_{w,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \right\}, \tag{D.11}$$

$$K_{w,t} = \left(h_t\ (w_t^*)^{\frac{\lambda_w}{\lambda_w-1}}\right)^{1+\sigma_L} + \beta \mathrm{E}_t \left\{ \xi_w(\chi_t) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \mu_z^*\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1} \right\}, \tag{D.12}$$

which satisfy

$$K_{w,t} = \left( \frac{1-\xi_w(\chi_t)\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \mu_{z,t}^*\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w(\chi_t)} \right)^{1-\lambda_w(1+\sigma_L)} \tilde{w}_t \frac{F_{w,t}}{\psi_L}. \tag{D.13}$$

The wage inflation equation is

$$\pi_{w,t} = \pi_t \mu_z^* \frac{\tilde{w}_t}{\tilde{w}_{t-1}}, \tag{D.14}$$

and the indexation rule is

$$\tilde{\pi}_{w,t} = (\pi_t^{target})^{\iota_w(\chi_t)} \pi_{t-1}^{1-\iota_w(\chi_t)}. \tag{D.15}$$

The efficiency condition for setting capital utilization is

$$r_t^k = \exp(\sigma_a(u_t-1))\, \bar{r}^k, \tag{D.16}$$

where the rental rate on capital is

$$r_t^k = \alpha \left( \frac{\Upsilon \mu_z h_t (w_t^*)^{\frac{\lambda_w}{\lambda_w-1}}}{u_t k_{t-1}} \right)^{1-\alpha} s_t. \tag{D.17}$$

The capital utilization costs are given by

$$a_t = \frac{\bar{r}^k\ (\exp(\sigma_a(u_t-1))-1)}{\sigma_a}, \tag{D.18}$$

and the capital adjustment costs are as follows

$$S_t = \exp\left[\sqrt{\frac{S''(\chi_t)}{2}} \left(\Upsilon \mu_z \frac{i_t}{i_{t-1}} - \Upsilon \mu_z\right)\right] + \exp\left[-\sqrt{\frac{S''(\chi_t)}{2}} \left(\Upsilon \mu_z \frac{i_t}{i_{t-1}} - \Upsilon \mu_z\right)\right] - 2 \tag{D.19}$$

The level of output is given by

$$y_t = p_t^{*\frac{\lambda_f}{\lambda_f - 1}} \left( \left( \frac{u_t k_{t-1}}{\Upsilon \mu_z} \right)^{\alpha} \left( h_t w_t^{*\frac{\lambda_w}{\lambda_w - 1}} \right)^{1-\alpha} - \phi \right) \tag{D.20}$$

and the marginal cost of production is

$$s_t = \left( \frac{r_t^k}{\alpha} \right)^{\alpha} \left( \frac{\tilde{w}_t}{1-\alpha} \right)^{1-\alpha} \tag{D.21}$$

The stationary household first-order conditions are

$$\lambda_{z,t} = \beta \mathrm{E}_t \left\{ \frac{\lambda_{z,t+1}}{\mu_{z,t}^* \pi_{t+1}} (1 + R_t) \right\}, \tag{D.22}$$

$$(1+\tau_c) \lambda_{z,t} = \frac{\mu_{z,t}^*}{\mu_{z,t}^* c_t - b c_{t-1}} - \beta \mathrm{E}_t \left\{ \frac{b}{\mu_{z,t}^* c_{t+1} - b c_t} \right\}, \tag{D.23}$$

$$\lambda_{z,t} = \lambda_{z,t} q_t \left( -S_t' \frac{i_t}{i_{t-1}} \Upsilon \mu_{z,t}^* + 1 - S_t \right) + \beta \mathrm{E}_t \left\{ \frac{\lambda_{z,t+1} q_{t+1} S_{t+1}'}{\Upsilon \mu_{z,t}^*} \left( \frac{\Upsilon \mu_{z,t}^* i_{t+1}}{i_t} \right)^2 \right\}, \tag{D.24}$$

where (D.22), (D.23), and (D.24) are with respect to risk-free bonds, consumption, and investment, respectively.

Regarding the entrepreneurs, the zero profit condition is as follows

$$\frac{q_t k_{t+1}}{n_{t+1}} \frac{R_{t+1}^k}{R_t} \left[ \Gamma_t(\omega_{t+1}) - \mu(\chi_t) G_t(\omega_{t+1}) \right] - \frac{q_t k_{t+1}}{n_{t+1}} + 1 = 0, \tag{D.25}$$

where

$$G_t(\omega_{t+1}) \equiv \int_0^{\omega_{t+1}} \omega dF_t(\omega), \qquad \text{and} \qquad \Gamma_t(\omega_{t+1}) \equiv \omega_{t+1} \left[ 1 - F_t(\omega_{t+1}) \right] + G_t(\omega_{t+1}). \tag{D.26}$$

The stationary entrepreneur first-order condition with respect to capital is given by

$$\begin{aligned} 0 = & \mathrm{E}_t \left\{ \frac{(1 - \Gamma_t(\omega_{t+1})) R_{t+1}^k}{R_t} \right. \\ & \left. + \frac{\Gamma_t(\omega_{t+1})'}{\Gamma_t(\omega_{t+1})' - \mu(\chi_t) G_t(\omega_{t+1})'} \left( \frac{R_{t+1}^k}{R_t} \left( \Gamma_t(\omega_{t+1}) - \mu(\chi_t) G_t(\omega_{t+1}) \right) - 1 \right) \right\}, \end{aligned} \tag{D.27}$$

where $\Gamma_t(\omega_{t+1})' = 1 - F_t(\omega_{t+1})$.

The return of capital for entrepreneurs is

$$R_t^k = \frac{(1-\tau_k) \left( r_t^k u_t - a_t \right) + (1-\delta) q_t}{\Upsilon q_{t-1}} \pi_t + \delta \tau_k, \tag{D.28}$$

The law of motion of entrepreneurial net worth is given by

$$n_{t+1} = q_t k_{t-1} \frac{\gamma}{\pi_t \, \mu^*_{z,t}} \left\{ R^k_t - R_{t-1} - \mu(\chi_t) \int_0^{\omega_t} \omega dF_{t-1}(\omega) R^k_t \right\} + n_t \left( \frac{R_{t-1}}{\pi_t \mu^*_{z,t}} \right) \gamma + W^e. \quad \text{(D.29)}$$

The monetary policy rule is as follows

$$\begin{aligned} \log\left(\frac{R_t}{R}\right) &= \rho_R \log\left(\frac{R_{t-1}}{R}\right) + \phi_\pi(\chi_t)(1-\rho_R)\frac{\pi^*}{R}\log\left(\frac{\pi_{t+1}}{\pi^*}\right) \\ &\quad + \phi_y(\chi_t)(1-\rho_R)\frac{\mu_z}{4R}\left(\frac{c_R \log\left(\frac{c_t}{c_{t-1}}\right) + i_R \log\left(\frac{i_t}{i_{t-1}}\right)}{(c_R + i_R)/(1-\eta_g)}\right), \end{aligned} \quad \text{(D.30)}$$

where $c_R$ and $i_R$ coefficients account for the share of consumption and investment in GDP.

The stationary resource constraint of the economy is

$$y_t = c_t + g_t + i_t + a_t \frac{k_t}{\Upsilon \mu^*_z} + d_t + \Theta \frac{(1-\gamma)}{\gamma}(n_{t+1} - W^e). \quad \text{(D.31)}$$

where $\Theta$ is the share of assets consumed by dying entrepreneurs and $d_t$ the monitoring costs, which are given by

$$d_t = \frac{\mu(\chi_t) G(\overline{\omega}_t) R^k_t q_{t-1} k_t}{\pi_t \mu^*_z}. \quad \text{(D.32)}$$

The law of motion for physical capital is

$$k_{t+1} = (1-\delta)\frac{k_t}{\Upsilon \mu^*_{z,t}} + \left(1 - S\left(\frac{i_t \mu^*_{z,t} \Upsilon}{i_{t-1}}, \chi_t\right)\right) i_t, \quad \text{(D.33)}$$

The law of motion for the uncertainty shock is as follows

$$\log \sigma_t = (1 - \rho_\sigma(\chi_t)) \log \sigma + \rho_\sigma(\chi_t) \log \sigma_{t-1} + \varepsilon_{\sigma,t}, \quad \text{(D.34)}$$

with $\varepsilon_{\sigma,t} = \text{normal}(\varepsilon_{\sigma,t} | 0, \sigma_\sigma(\chi_t))$.

## Appendix E. Results for alternative specifications of MS-DSGE model

Table 5. Posterior distribution, alternative specifications

| Coefficient | Description | Model | | | | |
|---|---|---|---|---|---|---|
| | | Benchmark | Taylor rule (1) CMR (2014) | Taylor rule (2) LWZ (2011) | Calvo CMR (2014) | Flexible prices and wages |
| $\phi_\pi(\chi_t = 1)$ | Taylor rule, inflation | 1.5073 [1.1040;1.9050] | – | – | 1.5410 [1.1137;1.8786] | 1.5151 [1.3090;2.0014] |
| $\phi_\pi(\chi_t = 2)$ | Taylor rule, inflation | 1.5260 [1.1464;1.9324] | – | – | 1.5028 [1.1582;1.9548] | 2.0699 [1.8671;2.3993] |
| $\phi_y(\chi_t = 1)$ | Taylor rule, output | 0.2464 [0.0785;0.4045] | – | – | 0.2453 [0.0672;0.3954] | 0.2504 [0.0939;0.4186] |
| $\phi_y(\chi_t = 2)$ | Taylor rule, output | 0.2514 [0.0907;0.4193] | – | – | 0.2546 [0.0906;0.4101] | 0.2896 [0.1290;0.4630] |
| $\xi_p(\chi_t = 1)$ | Calvo prices | 0.4874 [0.3113;0.6454] | 0.4910 [0.3204;0.6460] | 0.4892 [0.3373;0.6587] | – | – |
| $\xi_p(\chi_t = 2)$ | Calvo prices | 0.4848 [0.3231;0.6477] | 0.4895 [0.3304;0.6424] | 0.4850 [0.3317;0.6516] | – | – |
| $\xi_w(\chi_t = 1)$ | Calvo wages | 0.7926 [0.5738;0.8974] | 0.7993 [0.5879;0.9034] | 0.7954 [0.5863;0.8985] | – | – |
| $\xi_w(\chi_t = 2)$ | Calvo wages | 0.7546 [0.5458;0.8774] | 0.7633 [0.5505;0.8830] | 0.7536 [0.5574;0.8854] | – | – |
| $\iota_p(\chi_t = 1)$ | Price indexation | 0.5138 [0.2493;0.7464] | 0.5144 [0.2517;0.7590] | 0.5138 [0.2475;0.7587] | – | – |
| $\iota_p(\chi_t = 2)$ | Price indexation | 0.4913 [0.2453;0.7446] | 0.4933 [0.2528;0.7471] | 0.4909 [0.2348;0.7444] | – | – |
| $\iota_w(\chi_t = 1)$ | Wage indexation | 0.5218 [0.2538;0.7538] | 0.5320 [0.2732;0.7558] | 0.5260 [0.2733;0.7621] | – | – |
| $\iota_w(\chi_t = 2)$ | Wage indexation | 0.4383 [0.2236;0.7239] | 0.4478 [0.2240;0.7317] | 0.4347 [0.2292;0.7065] | – | – |
| $S''(\chi_t = 1)$ | Investment adj. costs | 0.6145 [0.2660;1.2983] | 0.5951 [0.2511;1.2911] | 0.6330 [0.3023;1.2687] | 0.6628 [0.2724;1.3411] | 1.8731 [1.4158;2.9182] |
| $S''(\chi_t = 2)$ | Investment adj. costs | 1.1692 [0.6192;1.8800] | 1.0973 [0.5978;1.8353] | 1.1750 [0.6657;1.8920] | 1.3688 [0.7537;1.9746] | 2.9305 [2.4572;3.7177] |
| $\mu(\chi_t = 1)$ | Monitoring cost | 0.2146 [0.1229;0.3271] | 0.2215 [0.1266;0.3547] | 0.2162 [0.1252;0.3609] | 0.2122 [0.1200;0.3397] | 0.1828 [0.1123;0.3257] |
| $\mu(\chi_t = 2)$ | Monitoring cost | 0.0602 [0.0399;0.1218] | 0.0608 [0.0373;0.1316] | 0.0605 [0.0395;0.1441] | 0.0550 [0.0366;0.1176] | 0.0673 [0.0500;0.1533] |
| $\rho_\sigma(\chi_t = 1)$ | Persistence shock | 0.5998 [0.3253;0.6967] | 0.6242 [0.3591;0.7066] | 0.5911 [0.3292;0.6972] | 0.6013 [0.3058;0.6805] | 0.5435 [0.2439;0.7613] |
| $\rho_\sigma(\chi_t = 2)$ | Persistence shock | 0.7580 [0.6913;0.8030] | 0.7711 [0.7122;0.8113] | 0.7569 [0.6994;0.8023] | 0.7538 [0.6846;0.8021] | 0.7227 [0.5869;0.7797] |
| $\sigma_\sigma(\chi_t = 1)$ | Std Dev shock | 0.3995 [0.3145;0.5439] | 0.4008 [0.3143;0.5441] | 0.3972 [0.3098;0.5311] | 0.4031 [0.3167;0.5459] | 0.3984 [0.2901;0.3904] |
| $\sigma_\sigma(\chi_t = 2)$ | Std Dev shock | 0.4549 [0.3258;0.5453] | 0.4591 [0.3122;0.5559] | 0.4531 [0.3057;0.5354] | 0.4526 [0.3147;0.5298] | 0.3339 [0.2340;0.3904] |
| $p_{11}$ | Transition matrix | 0.9847 [0.8600;0.9926] | 0.9884 [0.8901;0.9948] | 0.9842 [0.7706;0.9921] | 0.9869 [0.8491;0.9922] | 0.9800 [0.8119;0.9839] |
| $p_{22}$ | Transition matrix | 0.9891 [0.8823;0.9929] | 0.9874 [0.8693;0.9936] | 0.9893 [0.7706;0.9946] | 0.9902 [0.8798;0.9941] | 0.9999 [0.9947;0.9999] |
| $\kappa$ | Measurement VIX | 0.7888 [0.5744;1.3495] | 0.7382 [0.5468;1.4253] | 0.7971 [0.5772;1.5453] | 0.8196 [0.6173;1.5281] | 0.7198 [0.5086;1.6089] |
| Log Marginal Likelihood | | −22.9768 | −22.8045 | −23.0001 | −23.8766 | −45.0502 |

*Note:* Posterior modes and 90% probability intervals are reported. "CMR (2014)" stands for Christiano, Motto, and Rostagno (2014) and "LWZ (2011)" for Liu, Waggoner, and Zha (2011).